# Problem Set 1

*Due on Tuesday, February 21st*

**Important note:** Please start a new sheet of paper for each problem in the problem set. Write the names of the group members who contributed to the solution at the beginning of each problem.

## Problem 1

The basilisk lizard is an animal capable of running on water by supporting its weight by thrusting water downwards as it runs. The force exerted by thrusting water depends on the feet's sole area, $A$, the density of the fluid, $\rho$, and the speed of the animal, $U$.

**a)** Apply dimensional analysis to deduce an expression for the maximum weight that the basilisk lizard can support, $W$, as a function of $A$, $\rho$, and $U$.

**b)** The fastest person in the world is Asafa Powell. His 100 meter world record is 9.77 seconds. According to your result for part **a**, would he be able to run on water? (**Note:** In the expression deduced in part **a**, assume the constant of proportionality between the left hand side and the right hand side to be 1).

**c)** Do you think that the application of the expression deduced in part **a** to part **b** is correct? If not, why not?

# Problem 2

When the pressure in a liquid gets below vapor pressure, vapor bubbles are formed. This phenomenon is called cavitation. If these bubbles are then dragged to regions of higher pressure, they may suddenly collapse and cause damage to the container walls.
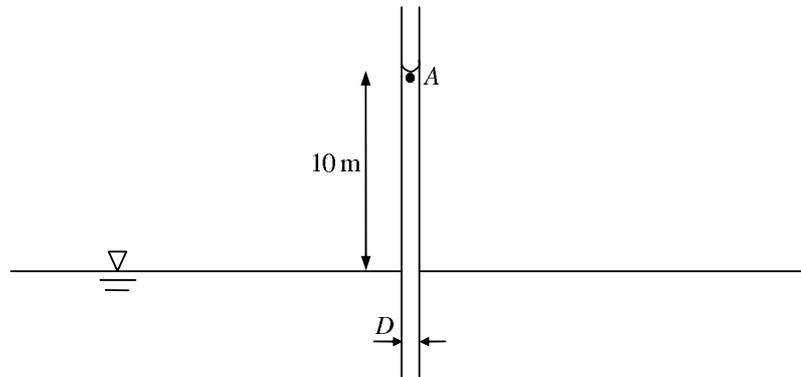


Figure 1: Capillary tube in Problem 2.

**a)** What diameter $D$ of clean glass tubing is required to get a rise of water at $20^oC$ due to capillary action of $10\ m$? (See Figure 1).

**b)** What is the pressure inside the tube at point $A$? What is the vapor pressure? Will cavitation occur? Assume $p_{atm} = 101.3\ kPa$.
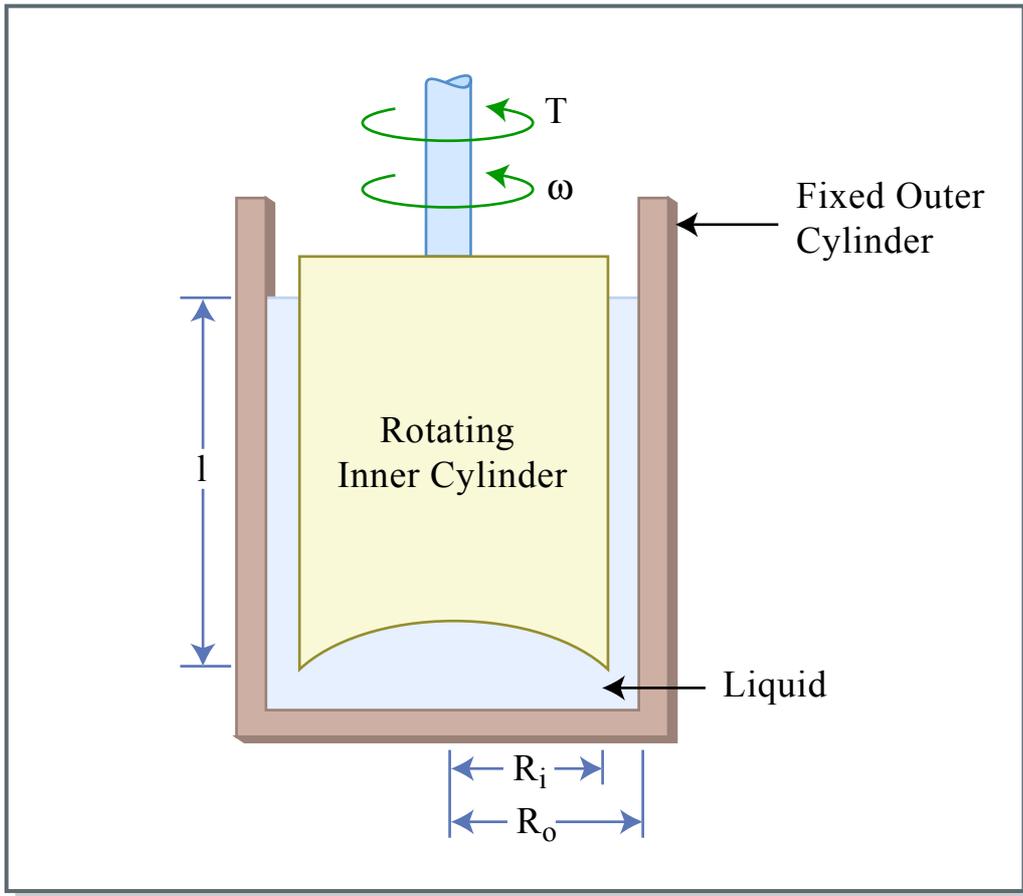
# Problem 3



Figure by MIT OCW.

The viscosity of liquids can be measured through the use of rotating-cylinder viscosimeter of the type shown in Figure 2. In this device, the outer cylinder is fixed and the inner cylinder is rotated with an angular frequency, $\omega$. The torque, $T$, required to maintain a constant $\omega$ is measured and the dynamic viscosity, $\mu$, is calculated from these two measurements. By neglecting end effects and assuming the velocity distribution in the gap to be linear, an equation relating $\mu$, $\omega$, $T$, $l$, $R_0$, and $R_i$ may be written as

$$T = \frac{2\pi R_i^3 l \omega}{R_0 - R_i} \mu.$$

**a)** Derive this equation and show that it is dimensionally correct.

The following torque-angular velocity data were obtained with a rotating-cylinder viscosimeter with $R_0 = 6.0\ cm$, $R_i = 5.9\ cm$, and $l = 25.0\ cm$:

| Torque $(N \cdot m)$ | 0.178 | 0.353 | 0.536 | 0.715 | 0.880 | 1.066 |
|---|---|---|---|---|---|---|
| Angular velocity (rad/s) | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 |

**b)** Give your best estimate of the viscosity, $\mu$.

**c)** Indicate the accuracy of this estimate.

**d)** Using Appendix B in Young et al.'s book, what fluid do you think was used if the temperature of the fluid was $15^oC$?

# Problem 4

A triangular plate of base $b$ and height $h$ is submerged in water, as shown in Figure 3. We consider four possible shapes of the plate, (i) to (iv), as represented in the figure.

**a)** For shape (i), calculate the pressure force exerted by the water on the surface $AB_1B_2$. Determine the position of the center of pressure. What is the relative position of the center of pressure with respect to the center of gravity?

**b)** For shape (ii), calculate the pressure force exerted by the water on the surface $A_1A_2B$. Determine the position of the center of pressure. What is the relative position of the center of pressure with respect to the center of gravity?

**c)** Using the results from **a** and **b**, repeat the exercise for shapes (iii) and (iv). (Note: You can answer this part without doing any further calculations!)
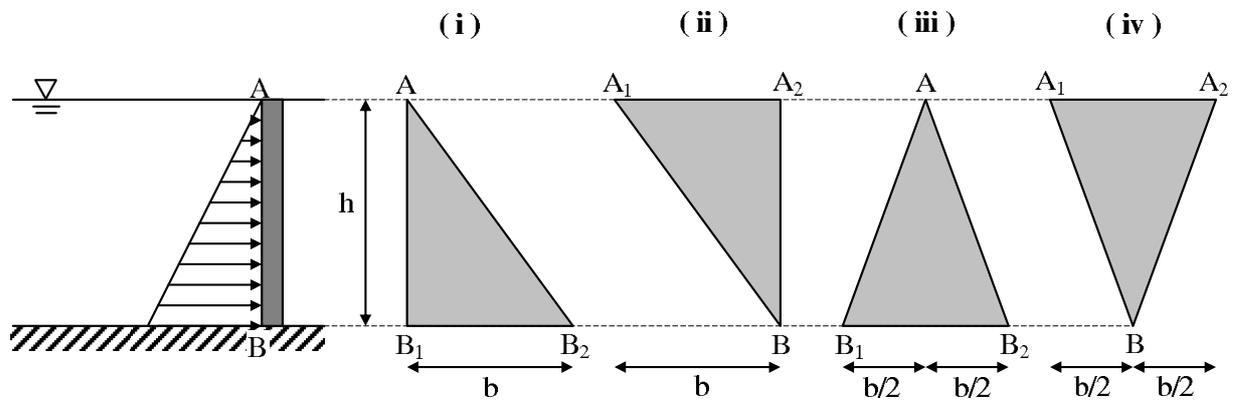


Figure 3: Submerged triangular plates in Problem 4.

# Problem 5

A centrifuge is a device that puts a fluid in rotation around a fixed axis, simulating a gravitational force. Centrifuges are used in biology and biochemistry to separate compounds of different molecular weights, in wastewater treatment to dry sludges, and in pilot and astronaut training to simulate the effect of accelerations above gravity. They could even be used in breakfast making to separate the cream from the milk.
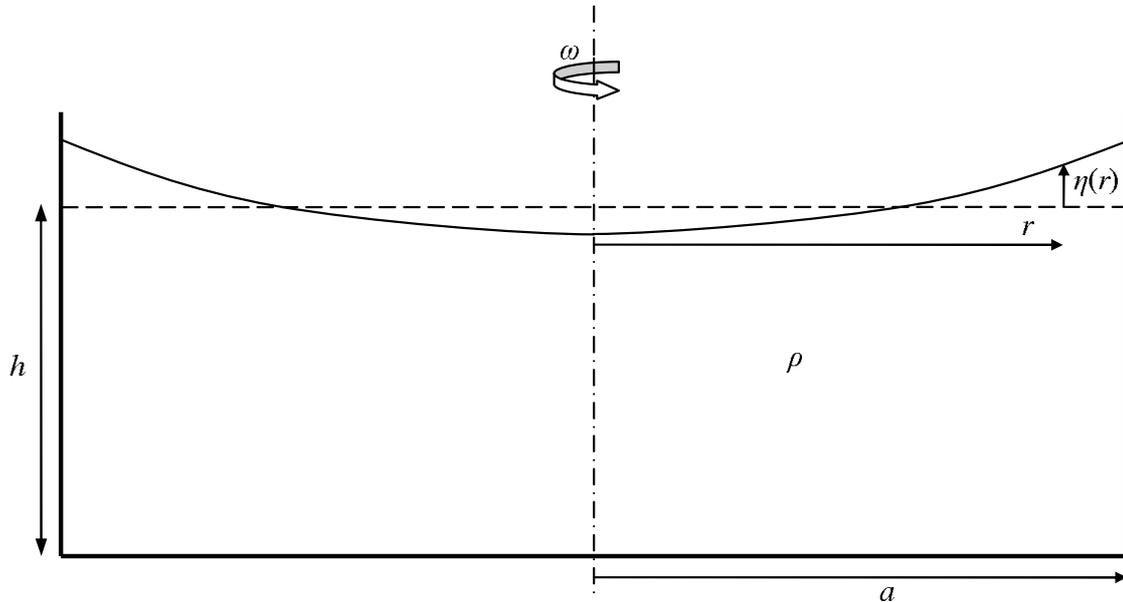


Figure 4: Centrifuge in Problem 5.

Consider the centrifuge in Figure 4. The interior container is a cylinder of radius $a$, which contains a liquid of density $\rho$. Initially, the liquid is at rest and its depth is $h$. The centrifuge starts to rotate and eventually reaches a uniform angular velocity $\omega$. Calculate the free surface elevation with respect to the still water level, $\eta(r)$, once the system is in steady state. If used to separate cream from milk, where would the cream end up?
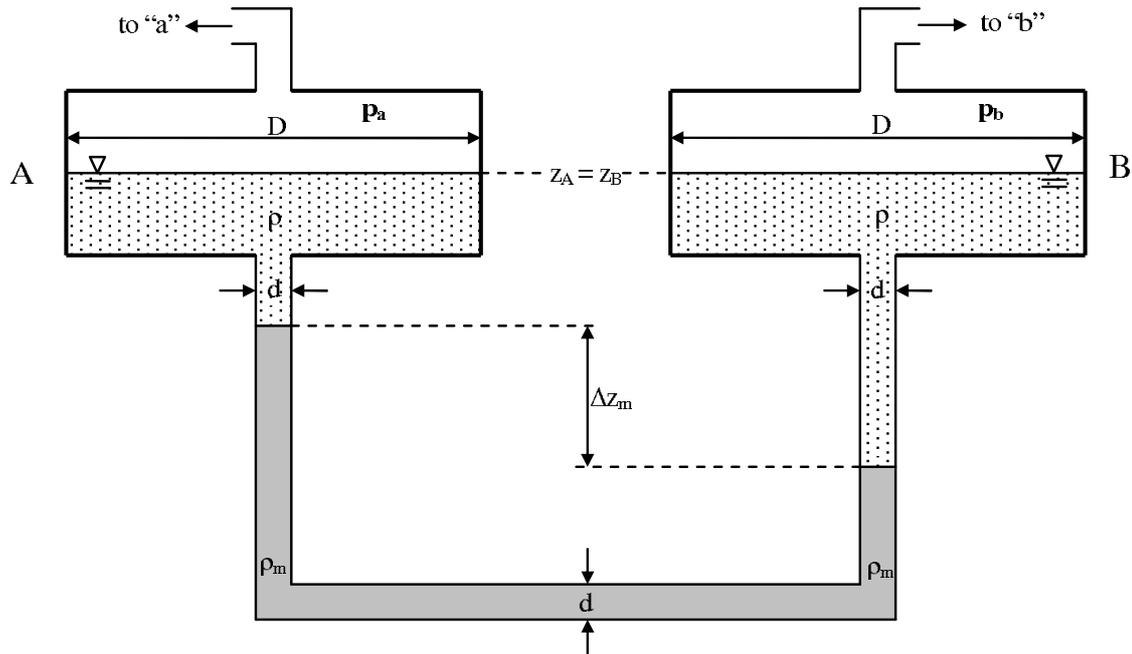
# Problem 6



Figure 5: Manometer in Problem 6.

Figure 5 shows a two-fluid manometer used to measure small pressure differences in gases, i.e., $p_a$ and $p_b$ are gas pressures. The two manometer chambers (A and B in the figure) contain the same fluid (a liquid of density $\rho$) and have circular horizontal cross sections of diameter $D$. The manometer chambers are connected through a small diameter, $d$, glass tube that contains the manometer fluid (a liquid of density $\rho_m > \rho$).

For a particular measurement the surfaces in the manometer chambers are **assumed** to be at the same elevation, i.e., $z_A = z_B$, whereas the elevation difference between the two surfaces of the manometer fluid, i.e., the actual manometer reading, is $\Delta z_m$ (see figure).

**a)** Derive an expression relating $\Delta z_m$, manometer reading, to the pressure difference $\Delta p = p_b - p_a$ and the two-fluid manometer densities $\rho_m$ and $\rho$.

**b)** If you expect to accurately measure a pressure difference $\Delta p = 1$ Pa, would you use a two-fluid manometer with fresh water and mercury ($\rho = 1000\ kg/m^3$ and $\rho_m = 13600\ kg/m^3$) or one with fresh and salt water ($\rho = 1000\ kg/m^3$ and $\rho_m = 1025\ kg/m^3$)? (Justify your answer in quantitative terms).

**c)** Why do you think it is important that the manometers chambers' diameters, $D$, are much larger than the manometer tube diameter, $d$? (**Hint:** Think of how this relates to the assumption that $z_A = z_B$).