7

Material Properties and Failure Phenomena

Up until now we have not said much about the way structures behave in the so-called "real world". Our focus has been on abstract concepts and pictures, with generous use made of mathematics in the formulation and solution of problems. There have been exceptions: We have talked about the properties of a rod, how we could measure the stress at which it would yield. We have talked about linear force/deflection relations and how we could measure the rod's *stiffness, k*. But our elaboration and application of the principles of equilibrium and compatibility of deformation have not required any reference to the things of the material world: Compatibility of deformation is a matter of the displacement of points, their absolute and relative displacement (when we talk about strain). Equilibrium of force and of moment concerns concepts that are just as abstract as displacements of points and rotations of infinitessimal line segments - if not more so. Constructing a free body diagram is an abstract, intellectual activity. No one goes out and actually cuts through the truss to determine the forces within its members.

In this chapter, we confront the world of different structural materials and their actual behavior. We want to know how the stiffness, k, depends upon the actual material constitution of our beam, or truss member, or concrete mix. We want to know how great a weight we can distribute over the beam or hang from the nodes of a truss before failure. We summarize our interests with two bullets:

- What properties characterize the behavior of a linear, elastic structural element? What is the general form of the stress/strain relations for an isotropic continuum?
- What conditions can lead to failure of a structure?

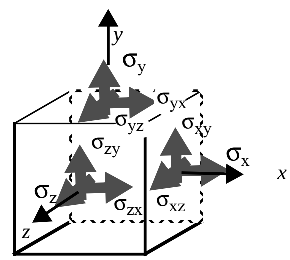
We begin with our elaboration of the constitutive relations for a continuum.

7.1 Stress/Strain Relations

We want to develop a set of stress/strain relations for a continuous body, equations which apply at each and every point throughout the continuum. In this we will restrict our attention, at least in this chapter, to certain type of materials namely *homogeneous, linear, elastic, isotropic bodies*.

- *Homogeneous* means that the properties of the body do not vary from one point in the body to another.

- *Linear* means that the equations relating stress and strain are linear; changes in stress are directly proportional to changes in strain (and the other way around, too).
- *Elastic* means that the body returns to its original, undeformed configuration when the applied forces and/or moments are removed.
- *Isotropic* means that the stress strain relations do not change with direction at a point. This means that a laminated material, a material with a preferred orientation of "grains" at the microscopic level, are outside our field of view, at least for the moment.

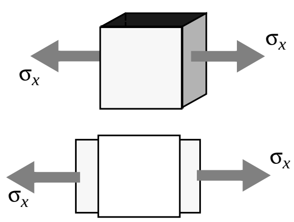


We have talked about *stress at a point*. We drew a figure like the one at the right to help us visualize the nature of the normal and shear components of stress at a point. We say the *state of stress is fully specified* by the normal components, σ_x, σ_y σ_z and the shear components $\sigma_{xy}=\sigma_{yx}$, $\sigma_{yz}=\sigma_{zy}$ $\sigma_{xz}=\sigma_{zx}$.

With these restrictions and a heavy dose of symmetry, we will be able to construct a set of stress/strain equations that will apply to many structural materials. This we do now, performing a sequence of thought experiments in which we apply to an element of stuff at a point each stress component in turn and imagine what strains will be engendered, which ones will not. Again, symmetry will be crucial to our constructions. We start by applying the normal stress component σ_x alone.

We expect to see some extensional strain ε_x. This we take as proportional to the normal stress we apply, in accord with the second bullet above; that is we set

$$\varepsilon_x = \sigma_x / E$$



In this we have made use of another bit of real experimental evidence in designating the constant of proportionality in the relationship between the extensional strain in the direction of the applied normal stress to be the *elastic, or Young's modulus*, *E*. We might not anticipate normal strains in the other two coordinate directions but there is nothing to rule them out, so we posit an ε_y and an ε_z.

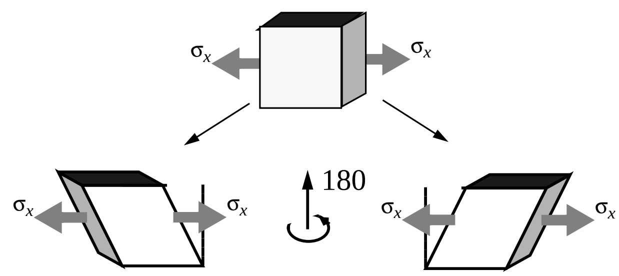
Now ε_y and ε_z, because of the indifference of the material to the orientation of the *y* and the *z* axis,– that is, from symmetry– must be equal. We can say nothing more on the basis of our symmetrical thoughts alone.

At this point we introduce another real piece of experimental data, namely that the material *contracts* in the *y* and *z* directions as it extends in the *x* direction due to the applied σ_x. We write then, for the strains due to a σ_x:

$$\varepsilon_y = \varepsilon_z = -\upsilon\varepsilon_x = -\upsilon \cdot \sigma_x / E$$

The ratio of the lateral contraction in the *y* and *z* directions to the extension in the x direction, the so called *Poisson's ratio* is designated by the symbol ν. We have encountered the magnitude of the elastic modulus E for 1020, cold rolled steel in the previous chapter. Poisson's ratio, ν is new; it takes on values on the order of one-quarter to one-half, the latter value characterizing an[1] incompressible material.

But what about the shear strains? Does σ_x engender any shear strains? The answer is no and here symmetry is all that we need to reach this conclusion. The sketch below shows two possible configurations for the shear strain γ_{xy}. Both are equally possible to an unbiased observer. But which one will follow the application of σ_x?



There is no reason why one or the other should occur.[2] Indeed they are in contradiction to one another; that is, if you say the one at the left occurs, I, by running around to the other side of the page, or more easily, by imagining the bit on the left rotated 180^o about a vertical axis, can obtain the configuration at the right. But this is impossible. These two dramatically different configurations cannot exist at the same time. Hence, neither of them is a possibility; a normal stress σ_x will not induce a γ_{xy}, or for that matter, a γ_{xz} shear strain.

By similar symmetry arguments, not provided here, we can rule out the possibility of a γ_{yz}.We conclude, then, that under the action of the stress component σ_x alone, we obtain only the extensional strains written out above.

Our next step is to apply a stress component σ_y alone. Now since the body is isotropic, it does not differentiate between the x and y directions. Hence our task is easy; we simply replace x by y (and y by x) in the above relationships and we have that, under the action of the stress component σ_y alone, we obtain the extensional strains

$$\varepsilon_y = \sigma_y / E \qquad\qquad \varepsilon_x = \varepsilon_z = -\upsilon(\sigma_y / E)$$

---

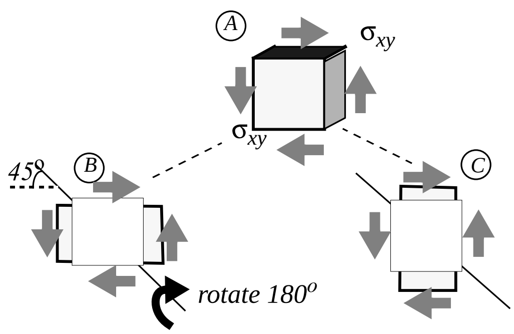
1. In fact, Poisson proved that, for an isotropic body, Poisson's ratio should be exactly one-quarter. We claim today that he was working with a faulty model of the continuum. For some relevant history on early nineteenth century developments in the continuum theories see Bucciarelli and Dworsky, *SOPHIE GERMAIN, an Essay in the Development of the Theory of Elasticity*
2. Think of the icon at the top as Buridan's ass, the two below as bales of hay.

The same argument applies when we apply the normal stress σ_z alone.

Now if we apply all three components of normal stress together, we will generate the extensional strains, and only the extensional strain.

$$\varepsilon_x = (1/E) \cdot [\sigma_x - \nu(\sigma_y + \sigma_z)]$$
$$\varepsilon_y = (1/E) \cdot [\sigma_y - \nu(\sigma_x + \sigma_z)]$$
and
$$\varepsilon_z = (1/E) \cdot [\sigma_z - \nu(\sigma_x + \sigma_y)]$$

One possibility remains: What if we apply a shear **stress**? Will this produce an extensional strain component in any of the three coordinate directions? The answer is no, and symmetry again rules. For example, say we apply a shear stress, σ_{xy}. The figure below shows two possible, shortly to be shown impossible, geometries of deformation which include extensional straining.



Now I imagine rotating the one on the left about an axis inclined at 45^o as indicated. I produce the configuration on the right. Try this with a piece of rectangular paper, a 3 by 5 card, or the like. But this is an impossible situation. The two configurations are mutually contradictory. A like cause, in this case a positive shear stress at the point, should produce a like effect. This is not the case. Hence, neither the deformation of *B* nor of *C* is possible.

There remains one further possibility: that a σ_{xy} generates an extensional strain in the *x* direction equal to that in the *y* direction. But this too can be ruled out by symmetry[3]. We conclude then that the shear strain σ_{xy}, or σ_{yz} or σ_{xz} for that matter, produces no extensional strains.

The expressions for the extensional strains above are not quite complete. We take the opportunity at this point to introduce another quite distinct cause of the

3. This is left as an exercies for the reader.

deformation of solids, namely a *temperature change*. The effect of a temperature change- say ΔT - is to produce an extensional strain proportional to the change. That is, for an isotropic body,

$$\varepsilon_{\text{x or y or z}} = \alpha \Delta T$$

The *coefficient of thermal expansion*, α, has units of $1/^{o}C$ or $1/^{o}F$ and for most structural materials is a positive quantity on the order of 10^{-6}. Materials with a *negative coefficient of expansion* deserve to be labeled *exotic*. They are few and far between.

The equations for the extensional components of strain in terms of stress and temperature change then can be written

$$\varepsilon_x = (1/E) \cdot [\sigma_x - \nu(\sigma_y + \sigma_z)] + \alpha \Delta T$$
$$\varepsilon_y = (1/E) \cdot [\sigma_y - \nu(\sigma_x + \sigma_z)] + \alpha \Delta T$$
and
$$\varepsilon_z = (1/E) \cdot [\sigma_z - \nu(\sigma_x + \sigma_y)] + \alpha \Delta T$$

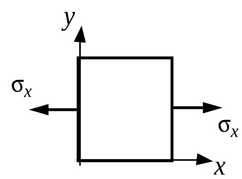
In the above, we ruled out the possibility of a shear stress producing an extensional strain. A shear stress produces, as you might expect, a shear strain. We state without demonstration that a shear stress produces only the corresponding shear strain. Furthermore, a temperature change induces no shear strain at a point. The remaining three equations relating the components of stress at a point in a linear, elastic, isotropic body are then.

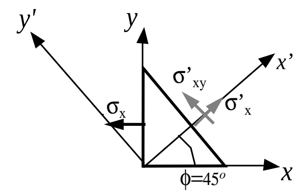
$$\gamma_{xy} = \sigma_{xy}/G$$
$$\gamma_{xz} = \sigma_{xz}/G$$
and
$$\gamma_{yz} = \sigma_{yz}/G$$

Recall that $\sigma_{xy} = \sigma_{yx}$. In these, G, *the shear modulus* is apparently a third elastic constant but we shall show in time that G can be expressed in terms of the elastic modulus and Poisson’s ratio according to:

$$G = \frac{E}{2(1 + \upsilon)}$$

To proceed, we consider a special instance of a case of plane stress, i.e., one in which the "z components" of stress at a point are zero. The special instance is shown in the figure, at the left.





We then consider the stress components acting upon a plane inclined at 45°. We relate the shear stress, σ'_{xy}, on the inclined plane to the normal stress σ_x through the appropriate transformation relation, namely:

$$\sigma'_{xy} = -\left[\frac{(\sigma_x - \sigma_y)}{2}\right] \cdot sin2\phi + \sigma_{xy} cos2\phi$$

With $\phi = 45^o$ this gives $\sigma'_{xy} = -\frac{\sigma_x}{2}$

Doing the same for the strain component, γ_{xy}',

$$\frac{\gamma'_{xy}}{2} = -\left[\frac{(\varepsilon_x - \varepsilon_y)}{2}\right] sin2\phi + \frac{\gamma_{xy}}{2} cos2\phi$$

gives $\frac{\gamma'_{xy}}{2} = -\left[\frac{(\varepsilon_x - \varepsilon_y)}{2}\right]$ Now we apply the stress strain relations

$$\varepsilon_x = (1/E) \cdot \sigma_x$$

$$\varepsilon_y = (1/E) \cdot [-\nu\sigma_x]$$

and

$$\gamma'_{xy} = \sigma'_{xy}/G$$

to this last relationship and obtain

$$\frac{\sigma'_{xy}}{2G} = -\frac{(1 + \upsilon)}{2E}\sigma_x$$

But from the transformation relationship between the stress components above, we know that $\sigma'_{xy} = -\frac{\sigma_x}{2}$. For these last two relationships to be consistent, we must have

$$\boxed{G = \frac{E}{2(1 + \upsilon)}}$$

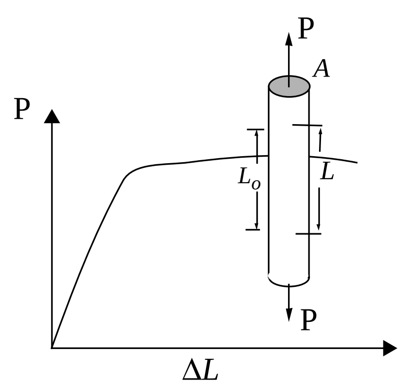
7.2 Properties of Ordinary Structural Materials

Contrary to my introductory remarks at the outset of this chapter, it seems we have proceeded abstractly in our exploration of the constitution of structural materials. The reason for this is that thinking things through is relatively cheap and inexpensive work compared to doing actual experiments in the real world. If we can figure out some ways in which our materials might, or must, behave by thinking abstractly about continuity and symmetry, about stress and strain, about rotation of axes - all the while making sure our analysis is logical and coherent - we have established a solid basis for fixing the behavior of real materials in the real world. This as long as our materials fit the assumptions of our model as set out in the bullets at the outset of this chapter[4]. Still, it doesn't give us the full picture, the full story; eventually we have to go into the lab to pull apart the actual stuff. To get our hands dirty, we explore how a bar in tension behaves.

Force/Deformation - Uniaxial Tension.

We have already said a few words about the failure of a truss member in tension – how a material like aluminum or steel will begin to *yield* or a more brittle material *fracture* when the tensile *stress* in the member becomes too large in magnitude. We want to say more now; in particular, we want to attend to the deformations that occur in a bar under uniaxial tension and look more closely at the mechanisms responsible for either *brittle fracture or the onset of yield*.

The tension test[5] is a standard test for characterizing the behavior of bars under uniaxial load. The test consists of pulling on a circular shaft, nominally a centimeter in diameter, and measuring the applied force and the *relative displacement* of two points on the surface of the shaft in-line with its axis. As the load P increases from zero on up until the specimen breaks, the relative distance between the two points increases from L_0 to some final length just before separation. The graph at the right indicates the trace of data points one might obtain for load P versus ΔL where



$$\Delta L = L - L_o$$

4. Recall how difficulties arise if there is a misfit - if our model is not appropriate as was the case concerning the behavior of the student in a chair on top of a table tilted up.
5. Standard tests for material properties, for failure stress levels, and the like are well documented in the American Society for Testing Materials, ASTM, publications. Go there for the description of how to conduct a tensile test.

Now, If we were to double the cross-sectional area, A, we would expect to have to double the load to obtain the same change in length of the two points on the surface. That indeed is the case, as Galileo was aware. Thus, we can extend our results obtained from a single test on a specimen of cross sectional area A and length L_0 to another specimen of the same length but different area if we plot the ratio of load to area, the *tensile stress*, in place of P.

Similarly, if, instead of plotting the change in length, ΔL, of the two points, we plot the stress against the *ratio of the change in length to the original length* between the two points our results will be applicable to specimens of varying length. The ratio of change in length to original length is just the *extensional strain*.

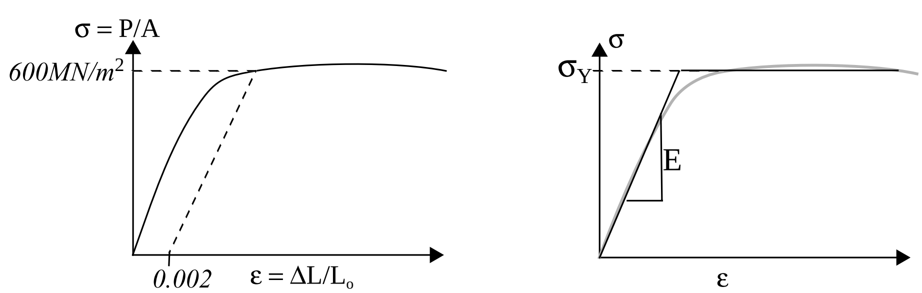
We, as customary, designate the *tensile stress* by σ and the *extensional strain* by ε; We assume that the load P is uniformly distributed over the cross sectional area A and that the relative displacement is “uniformly distributed” over the length L_0. Both stress and strain are rigorously defined as the limits of these ratios as either the area or the original length between the two points approaches zero. Alternatively, we could speak of an *average stress* over the section as defined by

$$\sigma \equiv P / A$$

and an *average strain* as defined by

$$\varepsilon \equiv (\Delta L) / L_o$$

The figure below left shows the results of a test of *1020, Cold Rolled Steel*. Stress, σ is plotted versus strain ε. The figure below right shows an abstract representation of the stress-strain behavior as *elastic, perfectly plastic* material.



Observe:

- The plot shows a region where the stress is proportional to the strain. The *linear* relation which holds within this region is usually written

$$\boxed{\sigma = E \cdot \varepsilon}$$

where E is the coefficient of *elasticity* of 1020CR steel - 30 x 10^6 lb/in^2.

- The behavior of the bar in this region is called *elastic*. **Elastic means that when the load is removed, the bar returns to its original, undeformed configuration.** That is L returns to L_0. There is no *permanent set*[6].
- The relative displacements of points — the strains in the elastic region — are very small, generally insensible without instruments to amplify their magnitude. To "see" a relative displacement of two points originally 100 *mm* apart when the stress is on the order of 400 *Mega Newtons*/m^2 your eyes would have to be capable of resolving a relative displacement of the two points of 0.2 *mm*! Strains in most structural materials are on the order of *tenths of a percent at most*.
- At some stress level, the bar **does not** return to its undeformed shape after removing the load. This stress level is called the *yield strength*. The yield strength defines the limit of elastic behavior; beyond the yield point the material behaves *plastically.* In most materials definition of a nominal value for the yield strength is a matter of convention. Whether or not the material has returned to its original shape upon removal of the load depends upon the resolution of the instrument used to measure relative displacement. The convention of using an offset relies upon the gross behavior of the material but this is generally all we need in engineering practice. In the graphs above, we show the *yield strength* defined at a *2% offset*, that is, as the intersection of the experimentally obtained stress-strain curve with a straight line of slope E intersecting the strain axis at a strain of 0.002. Its value is approximately 600 MN/m^2.
- Loading of the bar beyond the yield strength engenders very large relative displacements for relatively small further increments in the stress, σ. Note that the stress is defined as the ratio of the load to the *original area*; once we enter the region of plastic deformation, of *plastic flow*, the bar will *begin to neck down* and the cross sectional area at some point along the length will diminish. The *true stress* at this section will be greater than σ plotted here.
- For some purposes, it is useful to *idealize* the behavior of the material in tension as *elastic, perfectly plastic*; that is, the yield strength fixes the maximum load the material can support. This fantasy would have the material stretch out to infinite lengths once the yield strength was reached. For most engineering work, a knowledge of yield strength is all we need. We design to make sure that our structures never leave the elastic region.[7]

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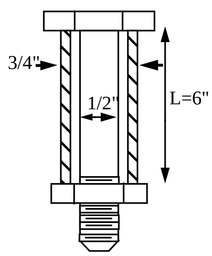
6. Note that *linear behavior* and *elastic behavior* are independent traits; one does not necessarily imply the other. A rubber band is an example of an elastic, non-linear material and you can design macro structures that are non-linear and elastic. Linear, inelastic materials are a bit rarer to find or construct.

7. On the other hand, if you are designing energy absorbing barriers, machine presses for *cold rolling or forming* materials and the like, plastic behavior will become important to you.

Exercise 7.1

A steel bolt, of 1/2 inch diameter, is surrounded by an aluminum cylindrical sleeve of 3/4" diameter and wall thickness, t= 0.10 in. The bolt has 32 threads/inch and when the material is at a temperature of 40^{o}C the nut is tightened one-quarter turn. Show that the uniaxial stresses acting in the bolt and in the sleeve at this temperature are σ_{bolt} *= 79 MN/m^2, and* σ_{sleeve} *= - 63 MN/m^{2}where the negative sign indicates the aluminum sleeve is in compression. What if the bolt and nut are cooled; at what temperature might the bolt become loose in the sleeve?*

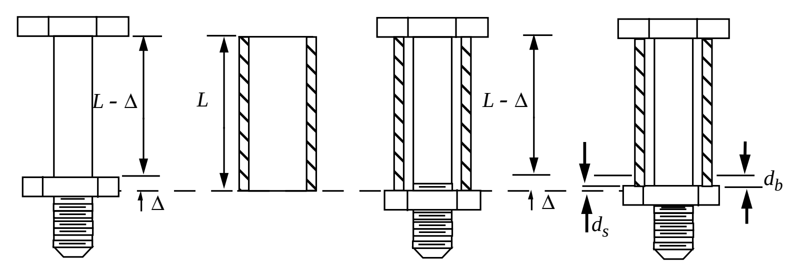
Compatibility of Deformation



Compatibility of Deformation is best assured by playing out a thought experiment about how the bolt and sleeve go from their initial unstressed, undeformed state to the final state. Think of the bolt and nut being separate from the sleeve. Think then of turning down the nut one quarter turn.

We show this state at the left, belo. Δ is the distance traveled in one quarter turn which, at 1/32 *inch*/*turn* is
Δ = 1/128 *in*

Next think of stretching the bolt out until we can once again fit the nut-bolt over the aluminum sleeve, the latter still in its undeformed state. This is shown in the middle figure below. Now, while stretching out the bolt in this way, replace the



Turn nut down *Stretch bolt - fit sleeve* *Release*

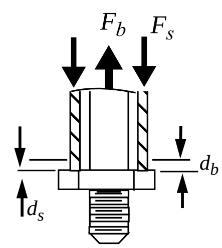
aluminum sleeve[8] then let go. The bolt will strive to return to its undeformed length – the behavior is assumed to be elastic – while the aluminum sleeve will resist contraction. The final state is shown at the right. The net result is that the steel bolt has extended **from its undeformed state** a distance $d_b$while the aluminum sleeve has contracted a distance d_s. We see from the geometry of these three

8. Since this is a *thought experiment* we don't have to worry about the details of this physically impossible move.

figures that we must have, for compatibility of deformation, $d_s + d_b = \Delta$ which is one equation in two unknowns.

Equilibrium

The figure at the right shows an isolation made by cutting through the bolt and the sleeve at some arbitrary section along the axis.



Note in this I have violated my usual convention. I have taken the force in the aluminum as positive in compression.

We let F_s be the **resultant** compressive force in the sleeve, the sum of the distributed loading around the circumference. F_b is the tensile force in the bolt. Like the carton-tie-down exercise, these two internal forces are *self equilibrating*; there are no external applied forces in the final state. We have $F_b - F_s = 0$

The normal stresses in the sleeve and the bolt are found assuming the resultant forces of tension and compression are uniformly distributed over their respective areas. Equilibrium then can be expressed as $\sigma_b \cdot A_b = \sigma_s \cdot A_s$ where the A's are the cross sectional areas of the bolt and of the sleeve.

Constitutive Relations

The constitutive relations are, for uniaxial loading, which is the case we have on hand,

$$\sigma_s = E_s \cdot (d_s / L) \qquad \text{and} \qquad \sigma_b = E_b \cdot (d_b / L)$$

We have then a total of four equations for four unknowns – the two displacements, the two stresses. Substituting the expressions for the stresses in terms of displacements into the equilibrium allows me to write

$$d_b = d_s \cdot (A_s E_s / A_b E_b)$$

which tells me the relative deformation as a function of the relative stiffness of the two material. If the sleeve is “softer”, the bolt deforms less... etc.

With this, compatibility gives me a way to solve for the displacements in terms of Δ. I obtain, letting

$$\beta = (A_s E_s / A_b E_b)$$

we have

$$d_s = \Delta \frac{1}{(1+\beta)} \qquad \text{and} \qquad d_b = \Delta \frac{\beta}{(1+\beta)}$$

Values for the stresses are found to be $\sigma_b = 79\ MN/m^2$ *and* $\sigma_s = 63\ MN/m^2$. (Note: compressive)

In computing these values, the elastic modulus for steel and aluminum were taken as 200 GN/m^2 and 70 GN/m^2 respectively. Observe that, though the steel experiences less strain, its stress level is greater in magnitude than that seen in the aluminum.

7.3 Table of Material Properties

The tension test is the standard test for determine *E,* the elastic or Young's modulus. Test that load a cylindrical specimen in torsion are used to measure the shear modulus *G*. Knowing *E* and *G*, Poisson's ratio may be obtained from the relationship we derived in the previous section.

What follows is a table giving the elastic properties and failure stresses in tension (and/or compression) for common structural materials. "Failure" means that ordinarily you want to design your structure such that you do not come close to this value under anticipated loading conditions.

The variety of materials included is meant to give the reader some idea of the range of property values of different kinds of structural materials. The values themselves are only meant to indicate orders of magnitude. In some cases, where the range of property values for a material is so large due to differences in composition or quality of its fabrication, a range has been shown. And certainly the table is not meant to be complete, nor should the values be used in detailed design work.

Material	Specific	Elastic Modulus		Failure Stress		α
	Gravity	10^6 psi	10^9 N/m^2	10^3 psi	10^6 N/m^2	10^{-6} /^{o}C
Al 2024-T3	2.7	10	70	60	400	23
Al 6061-T6	2.6	10	70	40	280	23
Al 7075-T6	2.7	10	70	80	550	23
Concrete	2.3	3	20	3 - 6[a]	20 - 40	7-12
High Strength	2.3	3 - 5	20 - 35	5-12	35 - 80	7-12
Copper	9	15	100	5	35	16
Glass Fiber	2.7	10	65	2000	15000	8
Iron (cast)	7	15	100	20-40	150-300	10
Steel High Strength	8	30	200	50-150	300-1000	14
Steel Structural	8	30	200	40-100	250-700	12
e.g., AISI C1020	8	30	200	85	600	12
Titanium	5	15	100	100	700	9
Wood (pine)	0.5	1.4	10	1	7	

a. In Compression

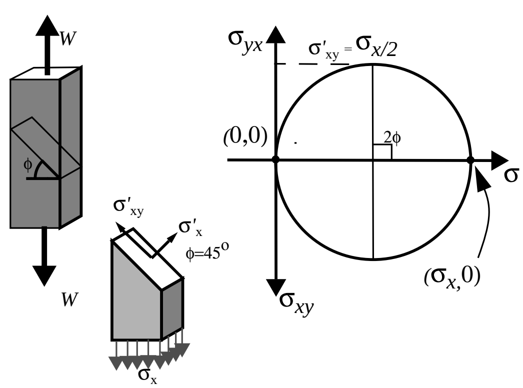
In all of this, failure stress is no unique number; in contrast to the elastic modulus, *E*, which is safe to take as a single value across different varieties and compositions of a material[9], the failure stress will vary all over the lot depending upon composition, care and means of fabrication. Compare the yield stress of cold rolled versus hot-rolled 1020 steel. Note too that one does not design to the failure stress but to a level significantly less than the numbers in the table. A *factor of safety* is always introduced to ensure that internal loads in the structure stay well below the failure level.

7.4 Failure Phenomena

Failure comes in different guises, in different sizes, colors, shapes, and with different labels. We have talked about *yielding, the onset of plastic flow* of *ductile* materials - materials which show relatively large, even sensible, deformations for relatively small increases in load once the material is loaded beyond its *yield strength*. If the excessive load is removed before complete collapse, the structure will not wholly return to its original, undeformed configuration.

Although it is the tension test that is used to fix the limit of elastic behavior and to define a *yield strength*, the mechanism for yielding is a shearing action of the material on a microscopic level. We have seen how a tensile stress in a bar can produce a shear stress on a plane inclined to the axis of the bar.

The figure shows the Mohr's Circle for a bar subjected to uniaxial tension. The maximum shear stress occurs on planes oriented at 45° to the axis of the bar. Its value is one-half the applied tensile stress. This then can be taken as a criterion for the onset of yield of a ductile material: Whenever the maximum shear stress at any point within any element of a structure exceeds one-half the applied tensile stress at the yield point in a tension test, the element will yield[10]



9. Concrete is an exception. Indeed it is difficult to find a linear portion of the stress/strain data taken from a specimen in compression.
10. In a general state of stress, the maximum shear stress is given by one-half the maximum difference of the (three) principle stresses. In the uniaxial tension test, this is just one-half the tensile stress as shown in the figure.

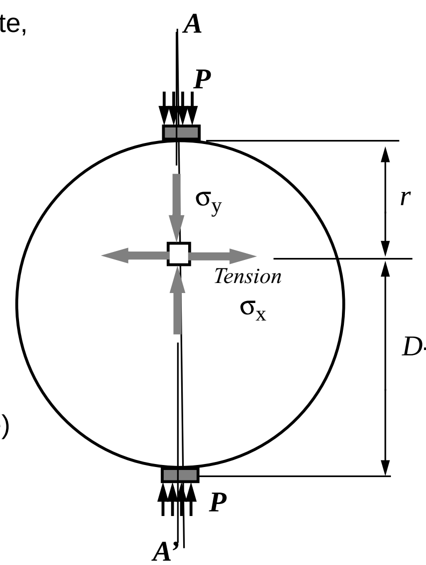
Not all materials behave as steel or aluminum or ductile plastics. Some materials are *brittle*. Load glass, cast iron, a brittle plastic, a carbon fiber, or concrete in a tension test and they will break with very little extension. They show insensible deformation all the way up to the fracture load. Recall the exercise in chapter 4 where we subjected a piece of chalk to torsion and how this generated a maximum tensile stress on a plane inclined at 45° to a generator on the surface of the chalk.

Metals generally carry a tensile or compressive load equally well. Concrete can not. Concrete generally can carry but one-tenth its allowable compressive load when subject to tension. A different sort of test is required to measure the tensile strength of concrete. (See the insert).

In an *indirect* tension test of concrete, a cylinder is loaded with a distributed load along diametrically opposed, sides of the cylinder. These line loads engender a uniform tensile stress distributed within the cylinder over the plane section A-A', bisecting the cylinder - except within the vicinity of the circumference. This tensile stress can be shown to be

$$\sigma_x = 2P/(\pi LD)$$

where L is the length (into the page) of the cylinder along which the load P is distributed, and D is the cylinder diameter.

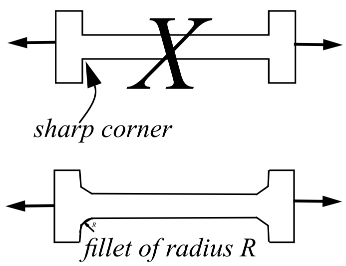


A compressive stress on planes orthogonal to A-A' is also engendered at each point. This can be shown to be equal to

$$\sigma_x = 2P/(\pi LD)\left[\frac{D^2}{r\,(D - r)} - 1\right]$$

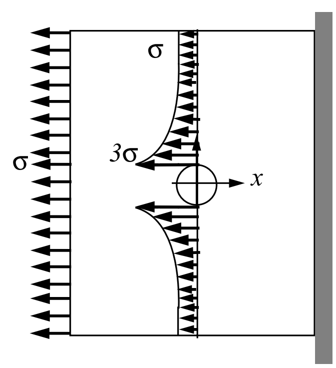
Values for the compressive and tensile failure stress levels depend upon the quality and uniformity of the composition of the material. Recall how we assumed our material was homogeneous. If this is not the case, then other features of the material must be taken into account. For example, in a composite material or structural element - a member made up of two different materials[11]- careful attention must be paid to the interface of the different materials out of which the member is made. We will consider some examples of composite structural elements in our chapter on the behavior of beams.

Another violation of homogeneity can prove disastrous: If the material, supposed continuous and uniform, contains an *imperfection* e.g., a microscopic crack, unseen by the naked eye even if on the surface, then all bets are off. (Rather all bets are on)! A crack can be the occasion for the magnification of stress levels in its immediate neighborhood.



This is one good reason why you see *fillets* at corners within a machined part. For example, one would never make a tensile test specimen in the shape shown at the top of the figure. The sharp corners where the two cylinders of differing diameters meet would engender a stress level at that junction significantly greater than the tensile stress found at the middle of the specimen, away from the junction.

A classic example of how a discontinuity within the interior of a material can act as a *stress raiser* is the solution obtained from the theory of elasticity for the stress field around a hole in a thin plate.



The figure at the right shows the effect of a circular hole on the stress component σ_x engendered in a thin plate subjected to a uniform tensile stress in the x direction. We show only the normal stress component σ_x acting on an x face which is a continuation of the vertical diameter of the hole. The hole gives rise to a stress concentration three times the magnitude of the uniformly applied tensile stress[12].

So far in our discussion we have been concerned with static loading conditions, i.e., we determine the internal stresses and static displacements due to loading - such as a dead weight. But some failure phenomena take time to develop. Even if there is no perceptible dynamic displacement or motion, materials age - like you and me - with the passage of time. Material properties and modes of failure also may depend upon temperature. What may be ductile at room temperature will be brittle at low temperatures. At high temperatures,

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11. Think of fiber reinforced skies, poles for pole vaulting, frames of tennis racquets and steel reinforced concrete slabs or beams.
12. Reference: Timoshenko and Goodier, *Theory of Elasticity*, McGraw-Hill, N.Y Third Edition 1970.

still well below the melting point, materials will *creep* – they will continuously deform at a constant load.

A material can fail in *fatigue*: Under continual cycling through tension then compression, a material will fail well below the yield strength or fracture stress witnessed in a tension test. Here the number of cycles may be large, on the order of thousands. But dynamic excitation of a structure can lead to failure in but a few cycles if the excitation drives a resonant mode of the system. Think of the Tacoma Narrows Bridge where aerodynamic excitation at the resonant frequency of a torsional mode of the structure led to ever increasing amplitude of vibration and the eventual spectacular collapse of the span. Before this failure event, aerodynamics and structures were the concerns of two different worlds[13].

Failure often occurs where you are not looking for it. If you spend all of your time doing models of structures made up of a large number of elements and focus solely on stress levels within the elements and pay little attention to the way the members are joined together and/or fixed to ground you are headed for trouble, for failure often occurs at joints. The Hyatt Regency walkway collapse is an unfortunate example.

Some failure phenomena are "macro"; they require more than the consideration of the state of stress at a point. *Buckling* is of this nature. We will study the buckling of beam-columns in the last chapter.

When we address the possibility of failure, inspection and testing become a necessity. Here we move from the abstract world of the theory of elasticity to the world of empirical data, manufacturers' specifications, of codes and traditional ways of fabrication and assembly. The problem is that in the design of the new and innovative structure, there is always the possibility that the codes and regulations and traditional ways of ensuring structural integrity do not exactly apply; something differs from the norm. If after a careful reading of existing code, any question remains, a full test program might be called for.

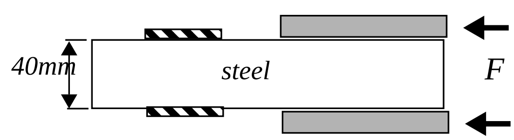
13. Except within the world of aerospace engineering where the coupling of aerodynamics and structural behavior were well attended to in the design of airfoils.

Design Exercise 7.1

A solid circular steel shaft of diameter 40 *mm* is to be fitted with a thin-walled circular cylindrical sleeve, *also made of steel*. In service the system is to serve as a stop, halting the motion of another fitted, but freely moving cylindrical tube whose inner radius is slightly larger than the outer radius of the solid shaft. The stopping sleeve is to remain in fixed location on the solid shaft for all axial loads less than some critical value of the force *F* shown in the figure. That is, for $F < 50kN$. If *F* exceeds this limit the sleeve is to frictionally break free and allow the sliding cylinder to continue moving. along the shaft.

It is proposed to fasten the sleeve to the shaft by means of a *shrink fit*. The initial inner radius of the sleeve is to be made slightly *less* than the initial outer radius of the shaft. The sleeve is then heated to a temperature not to exceed $\Delta Tmax = 250^oC$ so that its heat-treatment is not affected. The *hot* sleeve is then slipped over the shaft and positioned as desired. When the sleeve cools down, the radial *misfit* between the shaft outer radius and the sleeve's unstressed inner radius will generate sufficient mechanical interaction between the two so that the stopping and break-away functions can be fulfilled.

Size the sleeve.



7.5 Problems

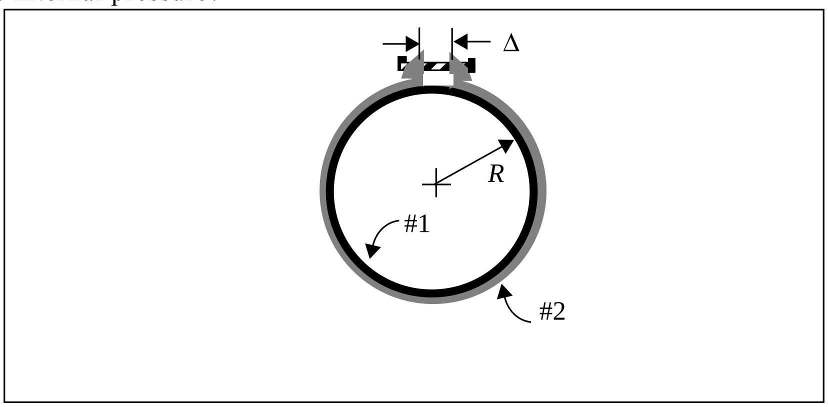
7.1 Hoop #1 is enclosed within hoop #2. The two are made of different materials, have different thicknesses but the same width (into the page). They are shown in their unstressed state, just touching. *Show that* after tightening the bolt at the top of the assembly and closing the gap, Δ, to zero, the stress in the outer hoop is tensile and has magnitude $F/(bt_1)$ while the stress in the inner hoop is compressive and has magnitude $F/(bt_2)$. In these t_1 and t_2 are the thicknesses,

$$F = k_1 k_2 \Delta/(k_1+k_2)$$

where

$$k_1 = (bt_1)E_1/L_1 \qquad \text{and} \qquad k_2 = (bt_2)E_2/L_2$$

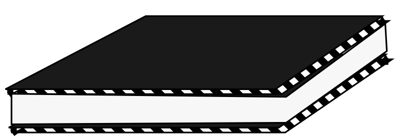
What if an internal pressure is applied to the inner hoop? When will the stress in the inner hoop diminish to zero? What will be the hoop stress in the outer hoop at this internal pressure?



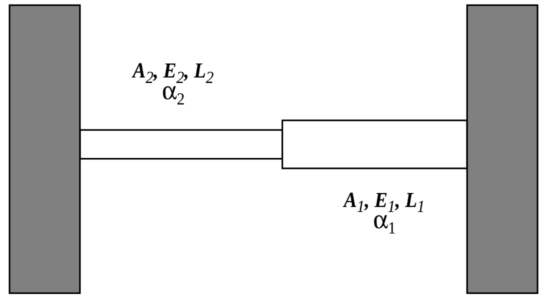
7.2 The thin plate is a composite of two materials. A quarter inch thick, steel, plate is clad on both sides with a thin (t_{al} = 0.005 in), uniform, layer of aluminum. The structure is stress-free at room temperature. *Show that* the stresses generated in the two materials, when the temperature changes an amount ΔT, may be approximated by

$$\sigma_{al} = (\alpha_{st} - \alpha_{al})E_{al}\,\Delta T/(1-\nu) \quad \text{and} \quad \sigma_{st} = -(2t_{al}/t_{st})(\alpha_{st}-\alpha_{al})E_{al}\Delta T/(1-\nu)$$

At what temperature will the clad plate begin to plastically deform? Where?



7.3 Two cylindrical rods, of two different materials are rigidly restrained at the ends where they meet the side walls. The system is subject to a temperature increase ΔT



How must their properties be related if the point at which they meet is not to move left or right?

If material #1 is steel and #2 is aluminum, what more specifically can you say?

E_1 = 200 GPa steel

E_2 = 70 GPa aluminum α_1 = 15 e-06 /^{o}C α_2 = 23 e-06 /^{o}C