1.050: Dimension Analysis (HW #2)

Due: September 19, 2007

MIT – 1.050 (Engineering Mechanics I)
Fall 2007
Instructor: Markus J. BUEHLER

Team Building and Team Work: We strongly encourage you to form Homework teams of three students. Each team only submits one solution for correction. We expect true team work, i.e. one where everybody contributes equally to the result. This is testified by the team members signing at the end of the team copy a written declaration that "the undersigned have equally contributed to the homework". Ideally, each student will work first individually through the homework set. The team then meets and discusses questions, difficulties and solutions, and eventually, meets with TA or instructor. Important: Specify all resources you use for your solution.

The following set of exercises is designed to train you in the use of dimensional analysis.

1. A vertical excavation: Consider a vertical excavation of a cohesive soil with strength σ_0.

(a) Using dimensional analysis, determine how deep can one cut into the soil before it collapses under the action of its own weight. What type of information does dimensional analysis provide at the end of the analysis? Comment.

(b) One O'Fifty Engineering Consultants have run an excavation experiment and determined the following properties for soil MIT1050:

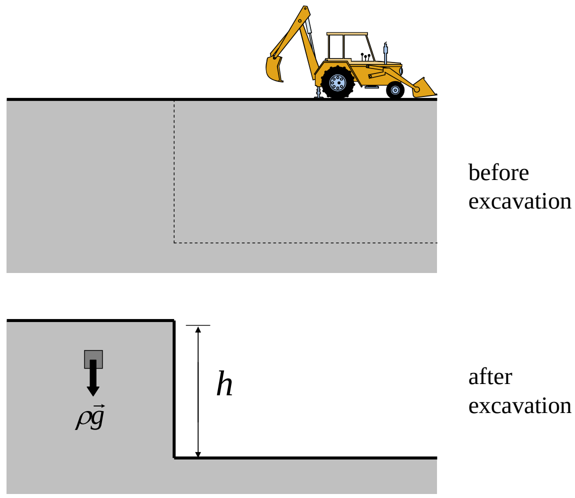
$$\rho = 2.4 g/cm^3$$
$$\sigma_0 = 10 kPa$$
$$h = 0.425 m$$

Using this data, could one determine explicitly the relation between the height of the excavation and the strength and weight of the soil wall?

(c) Your 1.050 TA run some soil tests and determined the following properties for two soil types:

$$clay:\ \sigma_0 = 30\ kPa,\ \rho = 1.7\ g/cm^3$$
$$sand:\ \sigma_0 = 3\ kPa,\ \rho = 1.4\ g/cm^3$$

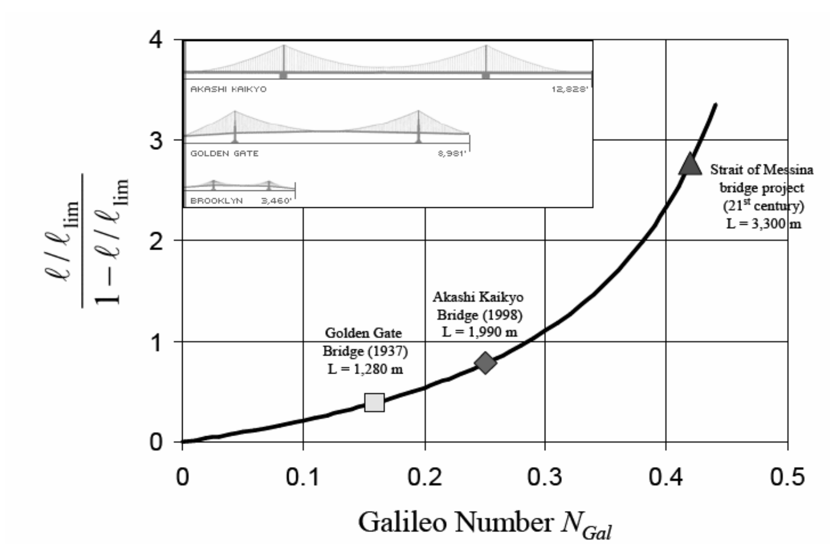
Based on your dimensional analysis results and the experimental data from One O'Fifty Engineering Consultants, determine the heights of excavation if you were working on those two types of soils. Comment on your results.



2. 'D-Analyzing' a record-breaking suspension bridge: Civil engineers are currently working on the design of an intercontinental bridge across the Red Sea, providing ground transportation between the Arabian Peninsula and the Horn of Africa. Upon successful completion of the project, it will surpass by more than 700 m the current record for the world's longest suspended span.

a) Read the article 'Record-breaking suspension span to link two continents' from *Civil Engineering Magazine*[*].
Comment on the challenges that engineers have to face in this project.

b) During recitation of week 2 (September 13-14), we will cover the application of dimensional analysis to bridge systems. Using the results derived in recitation (which can also be found in the class manuscript) and the relevant information about this new suspension bridge, calculate the Galileo Number and length factor $l/l_{\text{lim}}/(1-l/l_{\text{lim}})$ for the proposed suspension span. Plot your result in the following figure. Comment on the trend displayed in the figure.



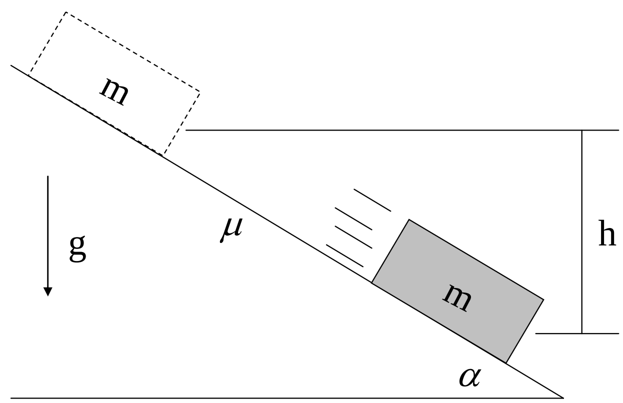
[*] Brown, J. L. (2007) Record-breaking suspension span to link two continents. *Civil Engineering*, 77 (8), August.

3. Velocity of a body in an included surface:

Problem text removed due to copyright restrictions. In addition to the variables shown in the diagram below, this problem considers the terminal velocity v and the frictional energy E_{frict}.

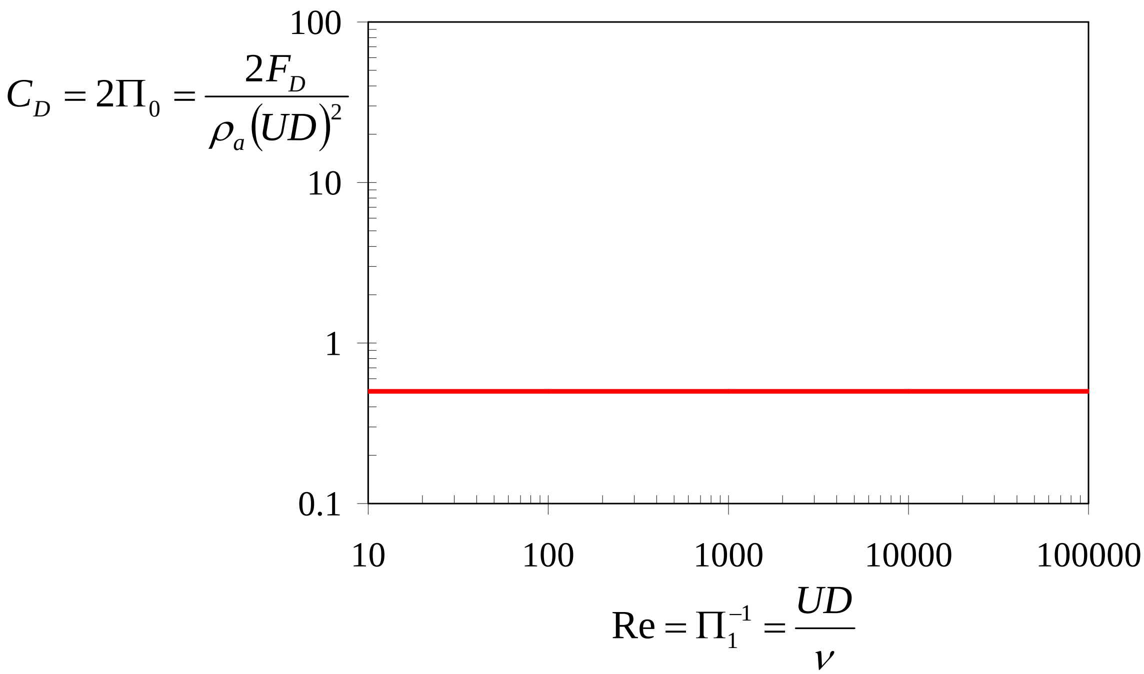
This is a typical physics problem. However this time, you will not resort to your physics problem-solving toolbox to analyze this problem. Instead, you will use your new dimensional analysis toolbox from 1.050[†].

(i) Before applying dimensional analysis, clearly establish which variables are dependent and which variables are independent.
(ii) Determine using dimensional analysis the possible valid relations among the given variables.
(iii) Could you explicitly calculate the terminal velocity by means of dimensional analysis alone? Comment.



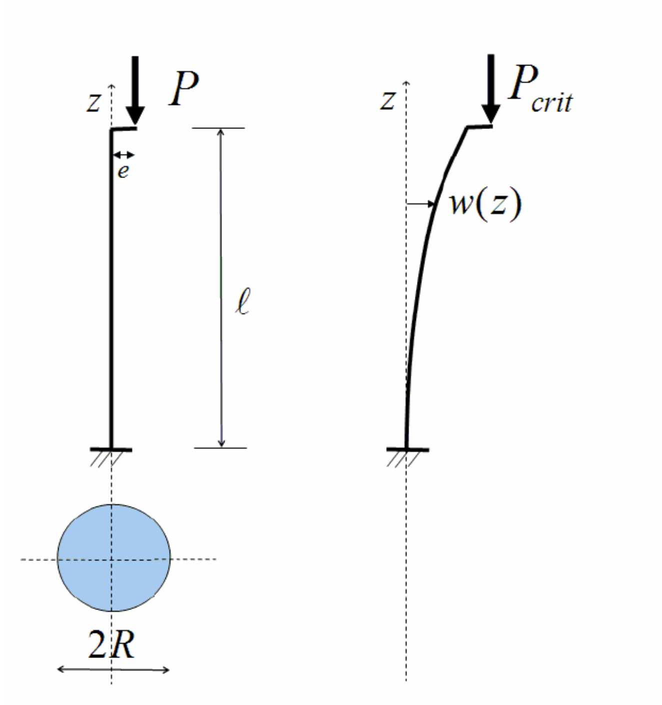
[†] Problem presented in *Szirtes, T. (2007) Applied Dimensional Analysis and Modeling, 2nd Ed. Elsevier, Amsterdam.*

4. Dimensional analysis in fluid mechanics: A spherical object of diameter $D = 10$ cm and density $\rho = 1{,}000$ kg/m^3 is vertically falling through air. Assuming the object started its trajectory from a high altitude, calculate its steady-state speed due to the presence of air drag. Use the Reynolds number – Drag Coefficient information from the enclosed figure for your calculations.



5. Euler - Buckling & the World Trade Center Towers: Following the destruction of the World Trade Center Towers on September 11, 2001, it has been reported that buckling of the columns had onset the fall of the towers. The focus of this exercise is to check this conjecture by means of a dimensional analysis. The first part of this exercise aims at deriving the governing equation of buckling; the second part then uses this result to make an informed analysis of the buckling of the columns of the WTC towers.

a) The buckling of columns is one critical stability problem in the design of beam-type structures. In contrast to the strength limit, the elastic stability problem of a column relates to the presence of imperfections. The elastic column buckling can be described as follows: consider a column of length l, and (for purpose of analysis) circular section of radius $R < l$. The column is made of an elastic material whose stiffness is defined by the Young's modulus E ‡. The column is subjected at its top to a force P. If the force were applied exactly in the column axis, the column would not buckle. However, imperfections are unavoidable. To consider those imperfections consider that the force was applied with an eccentricity of $e << R$ (see figure).



i. Buckling occurs when the applied force P reaches the buckling load P_{crit}. Formulate the problem, i.e., find the relevant parameters of the problem.

ii. The eccentric applied force causes a moment $M = Pe$, which leads to the deformation $w(z)$ of the column as sketched in the figure. As we will see later on in these lectures, the moment is related to this deformation by a linear moment curvature constitutive law:

$$M \propto ER^4 \frac{d^2 w}{dz^2}$$

‡ The concept of *stiffness* relates to the ability of the material to deform under application of loads. In 1.050, we will study this concept in depth, but at this time we will only introduce it within the framework of dimensional analysis.

(the term $d^2 w / dz^2$ is defined as the curvature). By choosing an appropriate base dimension system, show that the dimension function of the Young's modulus in the problem is:

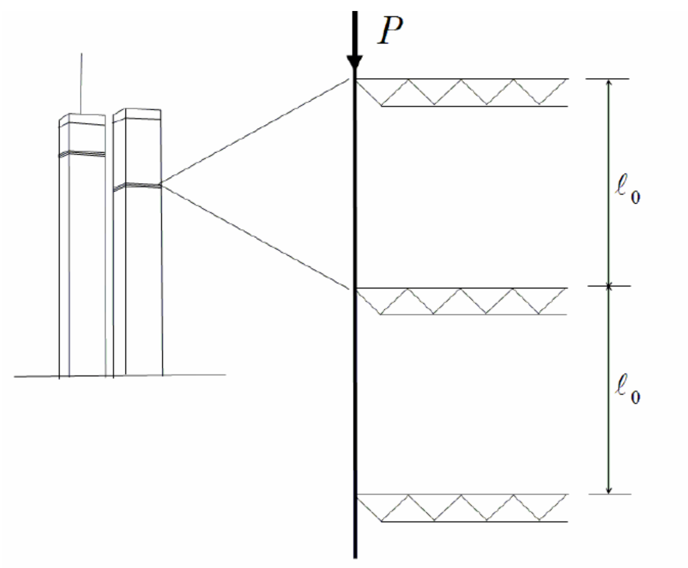
$$[E] = L_r^{-4} L_z^3 M T^{-2}$$

where L_r stands for the length base dimension in any direction perpendicular to the column span direction.

iii. From a dimensional analysis, determine the dimensionless relation for the buckling load. Show (and explain why?) that:

$$\lim_{e/R \to 0} \frac{P_{crit} l^2}{E R^4} = const.$$

b) In a real-life tall building, the columns are held by the connecting floors, which work as a horizontal bracing system (see figure). The columns in between two floors are designed so that the vertical load P is much smaller than the buckling load P_{crit}, defining the robustness $\gamma_0(z) = P_{crit} / P(z)$. In the light of the results so far obtained, explain the following:



i. The initial impact of the planes on the 110 floor World Trade Center towers may have destroyed one floor at the impact height, 96th floor in the North Tower and 80th floor in the South Tower. How did this change the robustness w.r.t. the design robustness γ_0?

ii. Using the analysis of part (i), develop an expression for the change in the robustness w.r.t. the design robustness γ_0 and the number of destroyed floors.

iii. It was reported that the ravaging fire close to the impact zone led to the destruction of several floor systems below the impact zone over time. If $\gamma_0(z=0)$ is the design robustness of the columns in the WTC at ground level, how many floors were destroyed before the tower collapsed? Make an estimate by considering the robustness of the WTC determined in the lecture to be $\gamma_0(z=0) \sim 20$. Distinguish North Tower vs. South Tower.
(Hints: $\gamma_0 = \gamma_0(z)$, since $P = P(z)$. Assume that $P(z)$ evolves linearly and it is a function of the dead weight of the structure).

iv. The North Tower was hit at 8:46 am above the 96th floor and remained erect until 10:28 am; the South Tower which was hit above the 80th floor at 9:03 am collapsed less than an hour later at 9:59 am. Explain why in a quantitative fashion.