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5.95J / 6.982J / 7.59J / 8.395J / 18.094J Teaching College-Level Science and Engineering
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Pictures help to accurately approximate $\ln n!$

$$3! = 3 \times 2 \times 1;$$

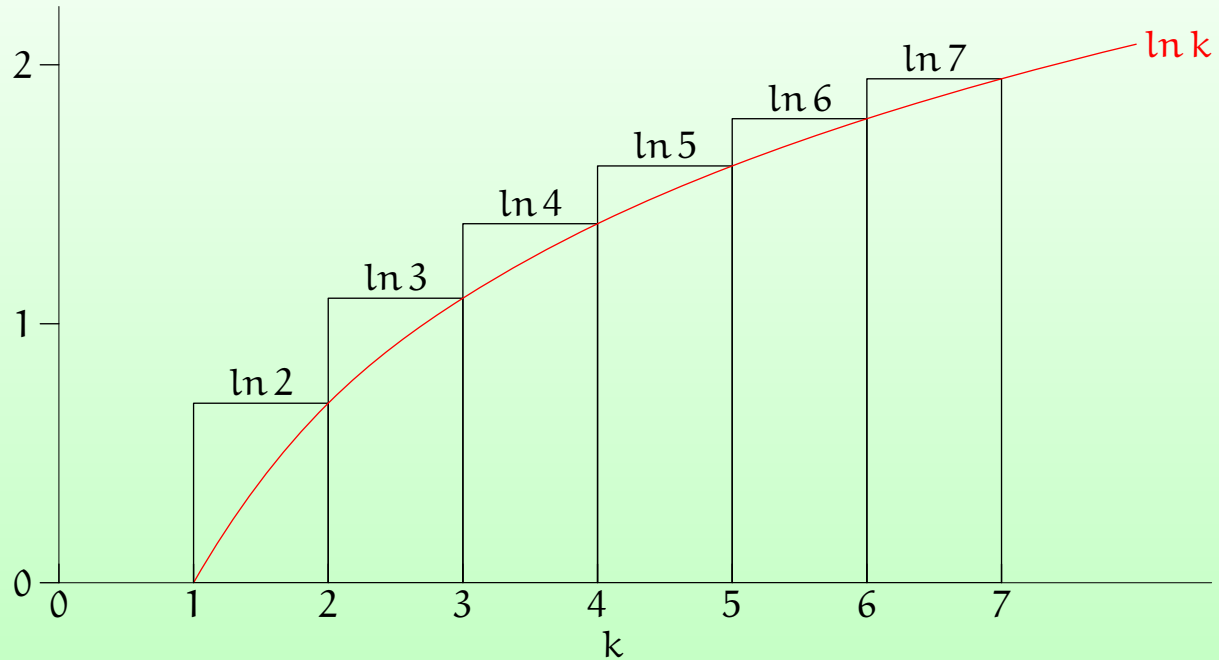
$$4! = 4 \times 3 \times 2 \times 1;$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1;$$

....

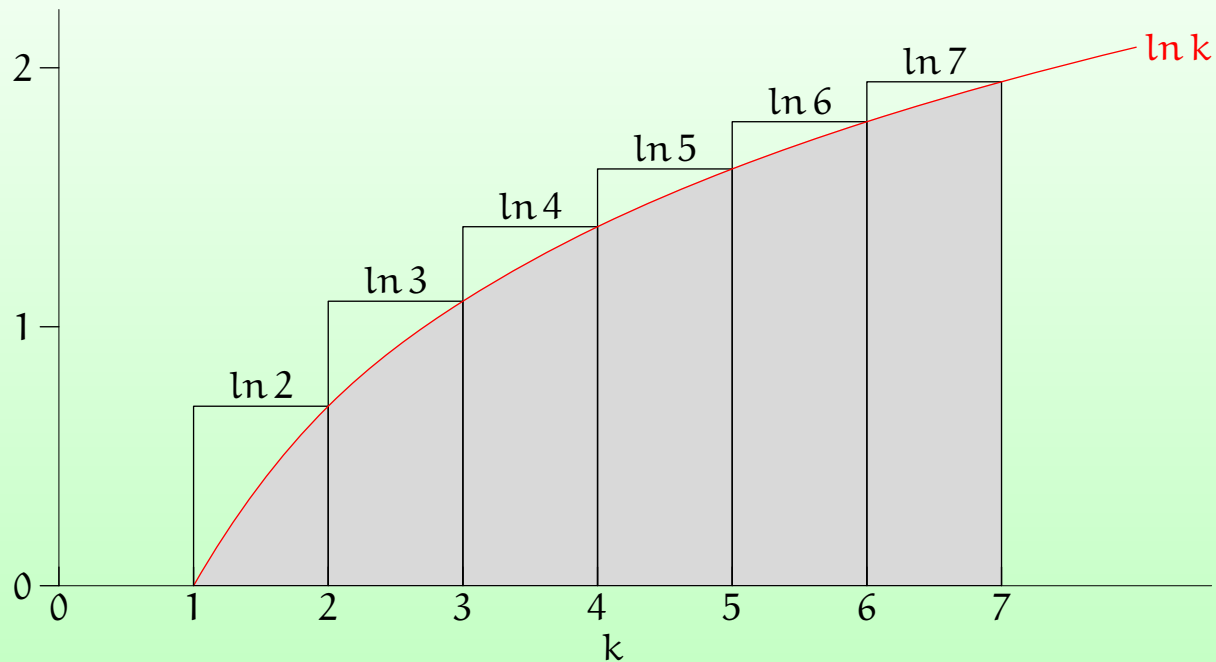
$n!$ is the most important function in statistical mechanics.

$\ln n!$ is the area of the rectangles



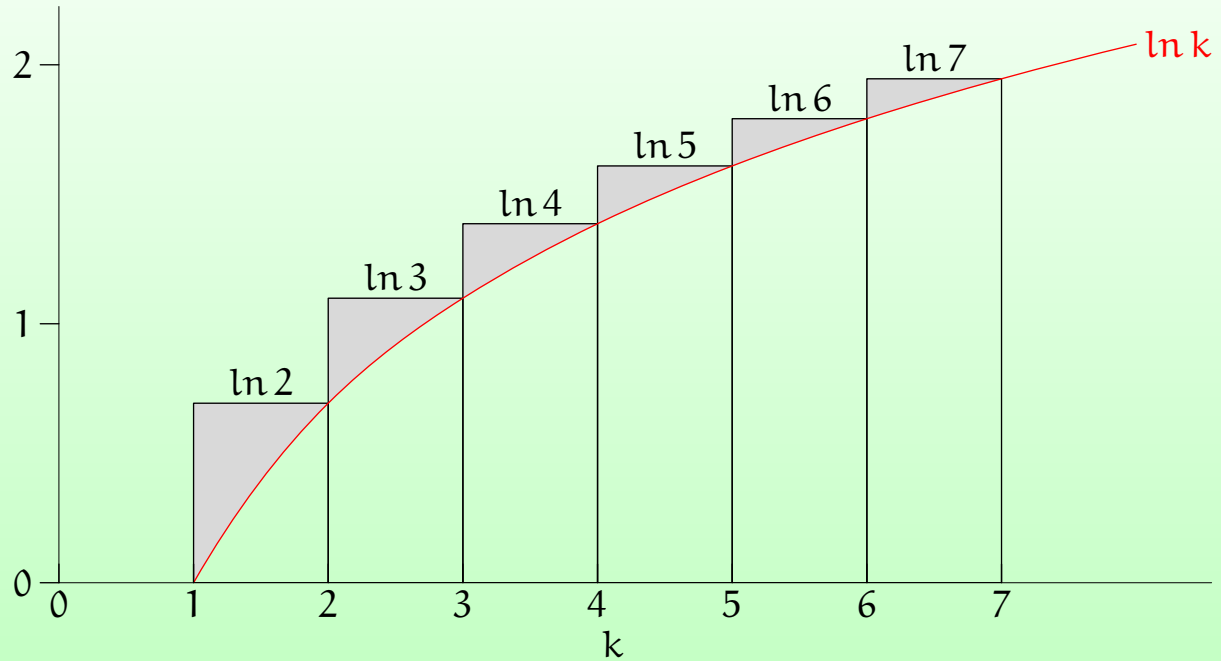
$$\ln n! = \sum_1^n \ln k$$

The area under $\ln k$ is the first approximation

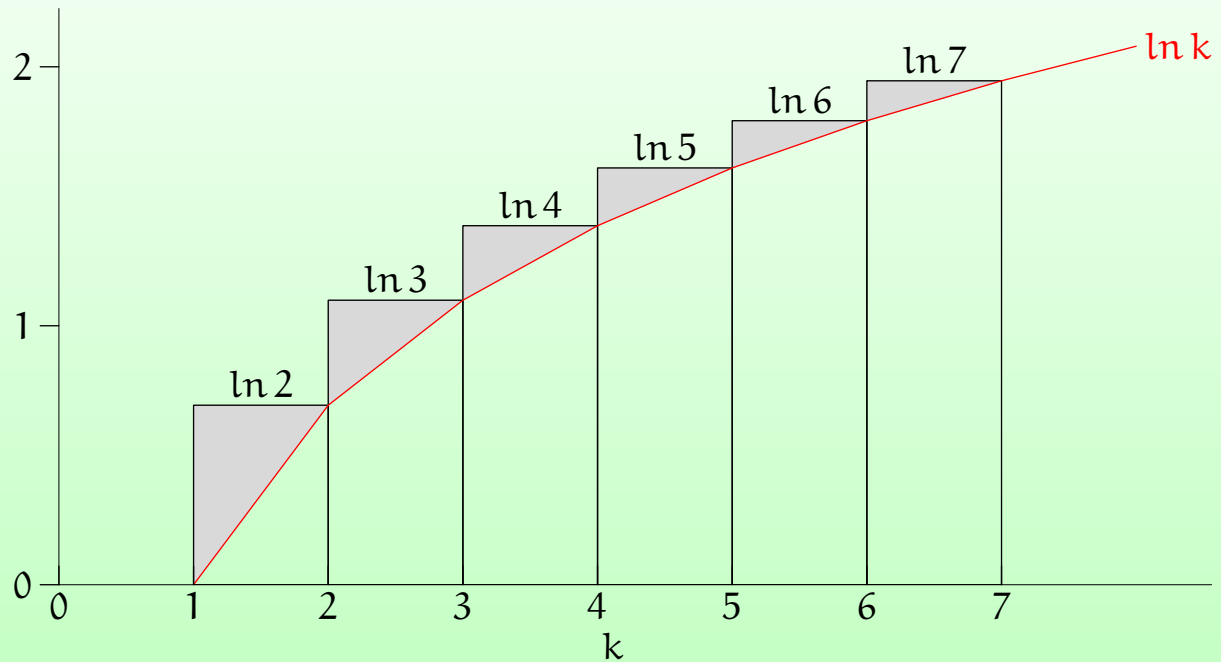


$$\ln n! \approx \int_1^n \ln k \, dk = n \ln n - n + 1$$

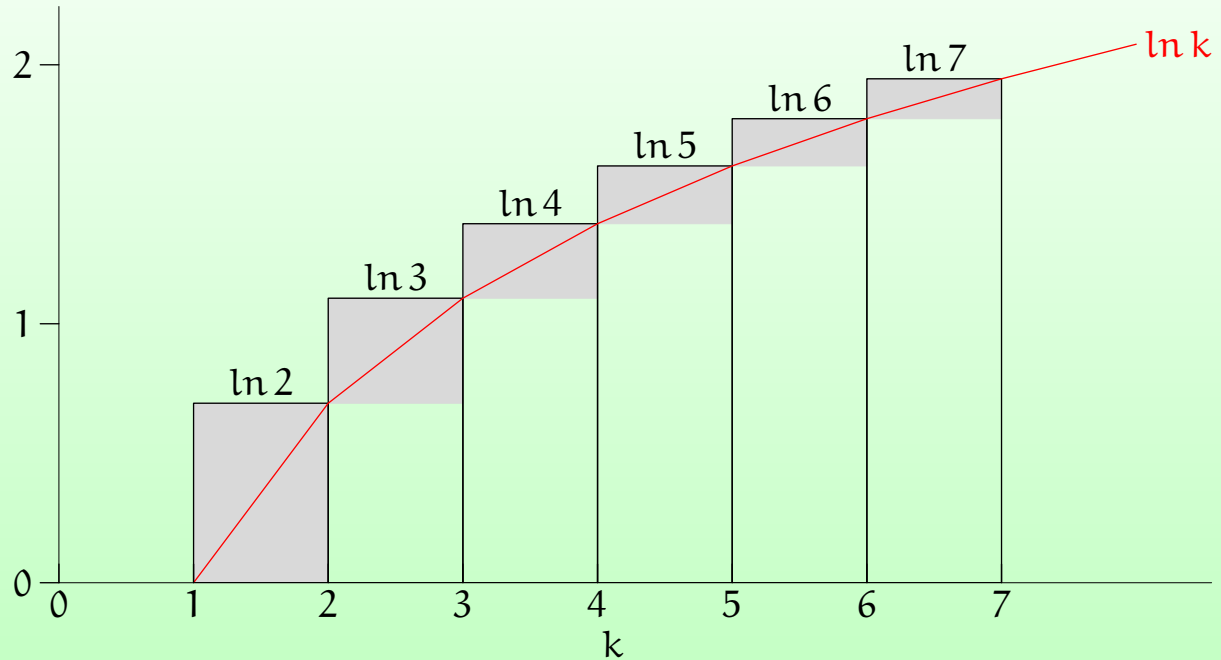
The error is the protruding pieces



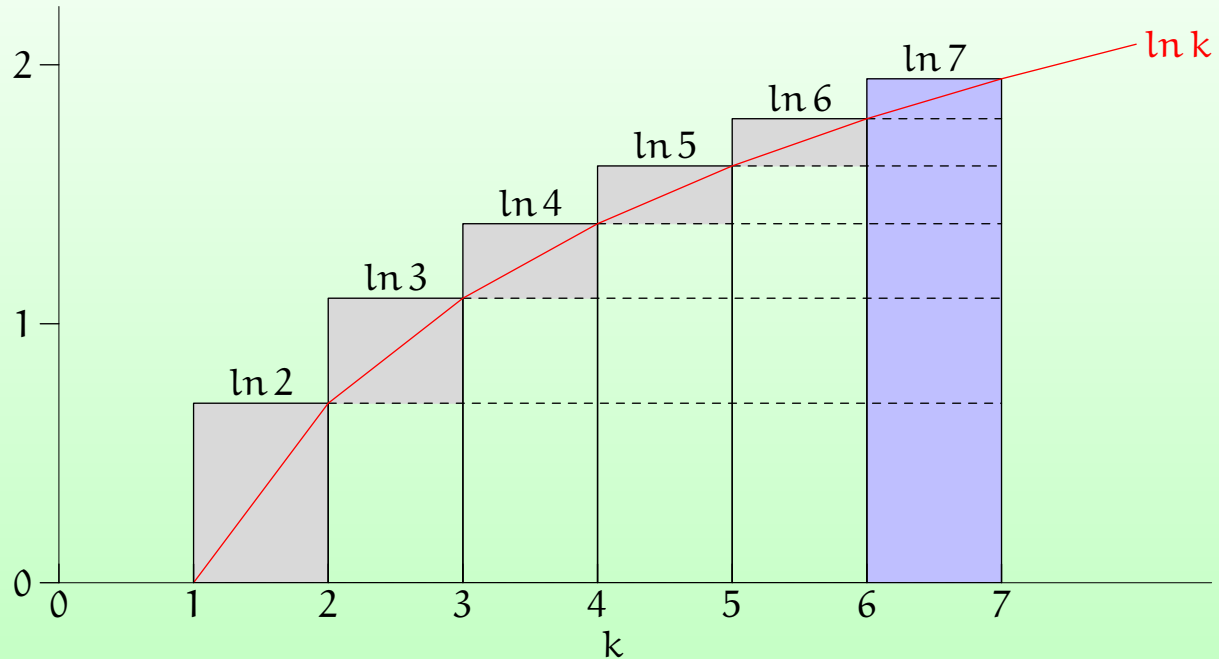
Each piece is almost a triangle



Doubling the 'triangles' makes them easier to add



The rectangles slide across and stack at the end

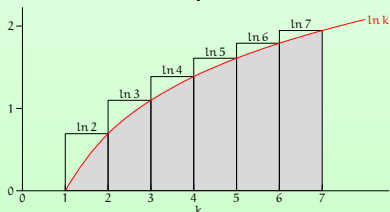


Sum of doubled protrusions = $\ln n$

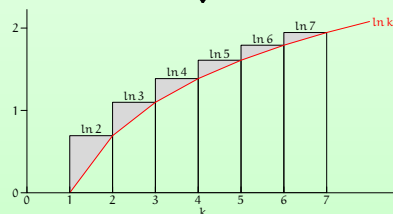
Combine the integral and approximated protrusion

$$\ln n! = \sum_{k=1}^n \ln k$$

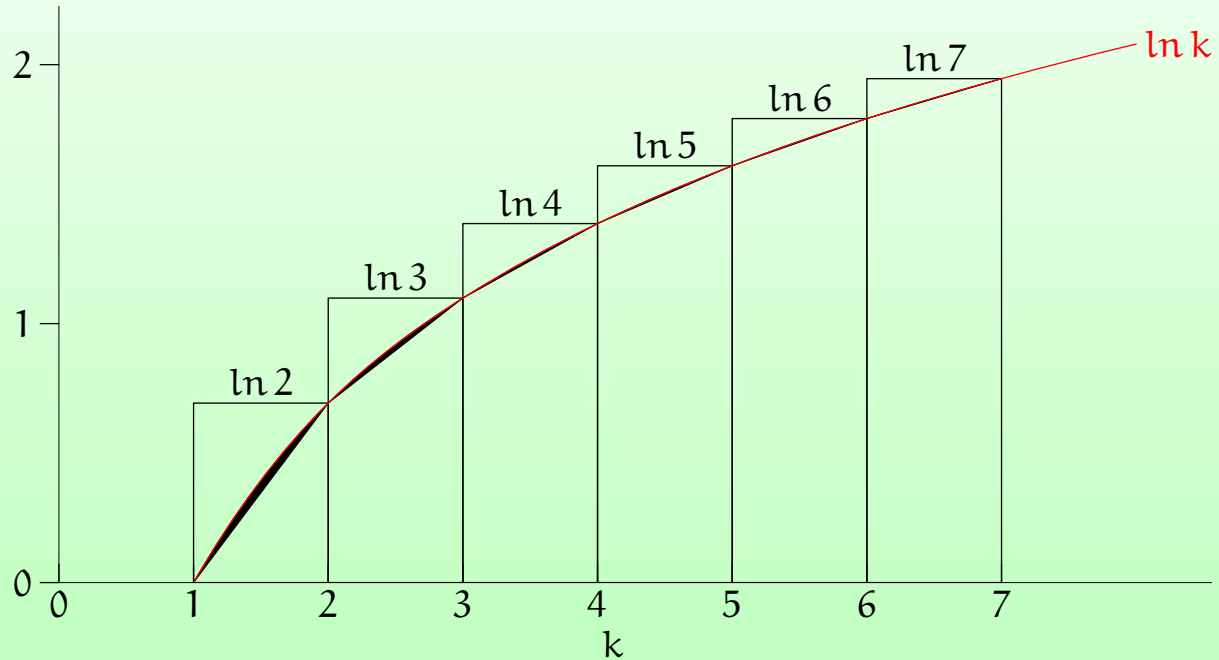
$$\approx \underbrace{n \ln n - n + 1}$$



$$+ \underbrace{\frac{1}{2} \ln n}$$



The preceding pictorial approximation ignores only a tiny region



Numerical calculation confirms the accuracy

Picture: $7 \times (\ln 7 - 1) + 1 + \frac{1}{2} \ln 7 = 8.594 \dots\dots$

Exact: $\sum_{k=1}^7 \ln k = 8.525 \dots\dots$

The approximation makes error of 0.07 in $\ln 7!$ (which results in a 7% error in $7!$).