5.73

Quiz 5 ANSWERS

$$\delta(a(x-b)) = \frac{1}{|a|}\delta(x-b).$$

$$\delta(g(x)) = \sum_i \left|\frac{dg(x_i)}{dx}\right|^{-1}\delta(x-x_i);\ x_i \text{ are zeroes of } g(x).$$

δ'(x-d) means derivative of δ-function evaluated at x=d.

Infinite box of width L: $E_n = \dfrac{n^2 h^2}{8mL^2}, k_n = \pm\dfrac{n\pi}{L}, n = 1,2,\ldots$

1.   f(x) = (x–3)(4x+8)

A.   Evaluate $\int \delta(x-d)f(x)dx$. $\boxed{= f(d) = (d-3)(4d+8)}$

B.   Evaluate $\int \delta'(x-d)f(x)dx$. $\boxed{= f'(d) = 8d-4}$

C.   Express $\delta(f(x))$ in terms of $\delta(x - x_i)$.

zeroes of $f(x)$ are $x = 3$ and $x = -2$

$\left.\dfrac{df}{dx}\right|_{x=3} = 20$, $\left.\dfrac{df}{dx}\right|_{x=-2} = -20$

$\delta(f(x)) = \dfrac{1}{20}\left[\delta(x-3) + \delta(x+2)\right]$

2.   Density of States.

A.   Compute $\dfrac{dn}{dk}$.

$= 1\left/\dfrac{dk}{dn}\right. = 1/(\pm\pi/L) = \pm L/\pi$

B.   Compute $\dfrac{dn}{dE}$.

$n = \left(\dfrac{8mL^2 E_n}{h^2}\right)^{1/2}$ $\dfrac{dn}{dE} = \dfrac{1}{2}\left(\dfrac{8mh^2}{h^2}\right)E_n^{-1/2}$

5.73 Quantum Mechanics I
Fall 2018