5.73

Quiz 26 ANSWERS

$$\mathbf{H}^{SO} = \frac{\zeta}{\hbar}\mathbf{L}\bullet\mathbf{S} = \frac{1}{2}\frac{\zeta}{\hbar}\left[\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2\right]$$
$$\mathbf{H}^{\text{Zeeman}} = -\gamma B_z(\mathbf{L}_z + 2\mathbf{S}_z)$$

Using ladders plus orthogonality, you should have obtained

$$\left|{}^2F_{7/2}, M_J = 5/2\right\rangle = \left(\frac{6}{7}\right)^{1/2}\left|{}^2F, M_L = 2, M_S = 1/2\right\rangle + \left(\frac{1}{7}\right)^{1/2}\left|{}^2F, M_L = 3, M_S = -1/2\right\rangle$$

and

$$\left|{}^2F_{5/2}, M_J = 5/2\right\rangle = -\left(\frac{1}{7}\right)^{1/2}\left|{}^2F, M_L = 2, M_S = 1/2\right\rangle + \left(\frac{6}{7}\right)^{1/2}\left|{}^2F, M_L = 3, M_S = -1/2\right\rangle.$$

A. Compute $E^{SO}\left({}^2F_{7/2}\right) = \left\langle {}^2F_{7/2}, M_J = 5/2\right|\mathbf{H}^{SO}\left|{}^2F_{7/2}, M_J = 5/2\right\rangle$.

F corresponds to $L = 3$, ${}^2F_{7/2}\ M_J = 5/2 = |J = 7/2, L = 3, S = 1/2, M_J = 5/2\rangle$

$$\left\langle {}^2F_{7/2}, M_J = 5/2\right|\mathbf{H}^{\text{SO}}\left|{}^2F_{7/2}, M_J = 5/2\right\rangle$$
$$= \left\langle \frac{7}{2}3\frac{1}{2}\frac{5}{2}\right|\frac{\hbar\zeta_{nf}}{2}\left[J(J+1) - L(L+1) - S(S+1)\right]\left|\frac{7}{2}3\frac{1}{2}\frac{5}{2}\right\rangle$$
$$= \frac{\hbar\zeta_{nf}}{2}\left[\left(\frac{7}{2}\right)\left(\frac{9}{2}\right) - 3(4) - \frac{1}{2}\left(\frac{3}{2}\right)\right] = \frac{\hbar\zeta_{nf}}{2}\left[\frac{63-48-3}{4}\right]$$
$$= \frac{\hbar\zeta_{nf}}{2}(3) = E^{\text{SO}}\left({}^2F_{7/2}, M_J = 5/2\right)$$

B. Compute $E^{SO}\left({}^2F_{5/2}\right) = \left\langle {}^2F_{5/2}, M_J = 5/2\right|\mathbf{H}^{SO}\left|{}^2F_{5/2}, M_J = 5/2\right\rangle$.

$$\left\langle {}^2F_{7/2}, M_J = 5/2\right|\mathbf{H}^{\text{SO}}\left|{}^2F_{7/2}, M_J = 5/2\right\rangle$$
$$= \frac{\hbar\zeta_{nf}}{2}\left[\frac{35}{4} - \frac{48}{4} - \frac{3}{4}\right] = -\frac{\hbar\zeta_{nf}}{2}(4) = E^{\text{SO}}\left({}^2F_{7/2}, M_J = 5/2\right)$$

C. Compute $E^Z\left({}^2F_{5/2},5/2\right)=\left\langle {}^2F_{5/2},M_J=5/2\left|\mathbf{H}^{Zeeman}\right|{}^2F_{5/2},M_J=5/2\right\rangle$.

$$\left\langle {}^2F_{7/2},M_J=5/2\left|-\gamma B_z\hbar\right|{}^2F_{7/2},M_J=5/2\right\rangle$$
$$=-\gamma B_z\hbar\left[\frac{1}{7}\left\langle {}^2F,M_L=2,M_S=s1/2\left|\mathbf{L}_z+2\mathbf{S}_z\right|{}^2F,M_L=2,M_S=1/2\right\rangle\right.$$
$$\left.+\frac{6}{7}\left\langle {}^2F,M_L=3,M_S=-1/2\left|\mathbf{L}_z+2\mathbf{S}_z\right|{}^2F,M_L=3,M_S=-1/2\right\rangle\right.$$
$$=-\gamma B_z\hbar\left[\frac{1}{7}(2+1)+\frac{6}{7}(3-1)\right]=-\gamma B_z\hbar\left(\frac{3}{7}+\frac{12}{7}\right)=-\gamma B_z\hbar\frac{15}{7}$$

D. Compute the off-diagonal matrix element:
$\mathbf{H}^Z_{5/2\,5/2;7/2\,5/2}=\left\langle {}^2F_{5/2},M_J=5/2\left|\mathbf{H}^{Zeeman}\right|{}^2F_{7/2},M_J=5/2\right\rangle$.

$$\mathbf{H}^{\text{Zeeman}}=-\gamma B_z\hbar\left[\frac{-6^{1/2}}{7}(2+1)+\frac{6^{1/2}}{7}(3-1)\right]=-\gamma B_z\hbar\left[\frac{6^{1/2}}{7}(2-3)\right]$$
$$=-\gamma B_z\hbar\left[\frac{6^{1/2}}{7}\right]$$

E. Use second order perturbation theory to derive the Zeeman tuning rate for the nominal ${}^2F_{5/2}$, $M_J = 5/2$ state. The Zeeman tuning rate is $\frac{dE}{dB_Z}$.

$$E^{(0)}\left({}^2F_{5/2},M_J=5/2\right)=E^{SO}\left({}^2F_{5/2}\right)+E^Z\left({}^2F_{5/2},5/2\right)$$
$$E^{(2)}\left({}^2F_{5/2},M_J=5/2\right)=\frac{\left|\mathbf{H}^Z_{5/2\,5/2;7/2\,5/2}\right|^2}{E^{SO}\left({}^2F_{5/2}\right)-E^{SO}\left({}^2F_{7/2}\right)}.$$

$$E^{(0)}\left({}^2F_{5/2},M_J=5/2\right)=\hbar\zeta_{nf}(-2)-\gamma B_z\hbar\left(\frac{15}{7}\right)$$
$$E^{(0)}\left({}^2F_{7/2},M_J=5/2\right)=\hbar\zeta_{nf}(3/2)-\gamma B_z\hbar\underbrace{\left[\frac{6}{7}(2+1)+\frac{1}{7}(3-1)\right]}_{20/7}$$
$$\mathbf{H}^{\text{Zeeman}}_{5/2,5/2;7/2,5/2}=\gamma B_z\hbar\left(\frac{6^{1/2}}{7}\right)$$
$$E^{(2)}\left({}^2F_{5/2},M_J=5/2\right)=\frac{\gamma^2B_z^2\hbar^2\left(\frac{6}{49}\right)}{\hbar\zeta_{n\ell}(-2-3/2)+\hbar\gamma B_z(-15/7+20/7)}$$