5.73

Quiz 24 ANSWERS

Pauli Matrices: $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \boldsymbol{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\boldsymbol{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \boldsymbol{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

A. What are the eigenvalues of $\boldsymbol{\sigma}_x$, $\boldsymbol{\sigma}_y$, and $\boldsymbol{\sigma}_z$?.

σ_x:	$E^2 - 1 = 0$	$E = \pm 1$ (E is for eigenvalue)
σ_y:	$E^2 - 1 = 0$	$E = \pm 1$
σ_z:	$(1-E)((-1-E) = 0$	$(-1+E^2) = 0 \quad E = \pm 1$

B. Let $M = \begin{pmatrix} 1 & 3\cos\omega t \\ 3\cos\omega t & 4 \end{pmatrix}$. Find the trace of

(i) **MI**

MI = M

Trace M = 1 + 4 = 5

(ii) $\mathbf{M}\boldsymbol{\sigma}_x$

$$\begin{pmatrix} 1 & 3\cos\omega t \\ 3\cos\omega t & 4 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3\cos\omega t & 1 \\ 4 & 3\cos\omega t \end{pmatrix}$$

$$\text{Trace}(\mathbf{M}\sigma_x) = 3\cos\omega t + 3\cos\omega t$$
$$= 6\cos\omega t$$

(iii) $\mathbf{M}\boldsymbol{\sigma}_y$

$$\begin{pmatrix} 1 & 3\cos\omega t \\ 3\cos\omega t & 4 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = 3\cos\omega t - 3i\cos\omega t$$
$$= 0$$

(iv) $\mathbf{M}\boldsymbol{\sigma}_z$

$$\begin{pmatrix} 1 & 3\cos\omega t \\ 3\cos\omega t & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -3\cos\omega t \\ 3\cos\omega t & -4 \end{pmatrix}$$

$$\text{Trace}(\mathbf{M}\sigma_z) = 1 = 4 = -3$$

(over)

C. Let $\rho(t)=\frac{1}{5}\mathbf{M}$. $\mathbf{M}=\begin{pmatrix} 1 & 3\cos\omega t \\ 3\cos\omega t & 4 \end{pmatrix}$ Consider the vector

$$a_x=\frac{1}{2}Tr\left(\rho\sigma_x\right)=\frac{1}{10}\mathrm{Tr}\begin{pmatrix} 1 & 3\cos\omega t \\ 3\cos\omega t & 4 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}=\frac{1}{10}(1+3\cos+3\cos)=\frac{1}{10}+\frac{3}{5}\cos\omega t$$

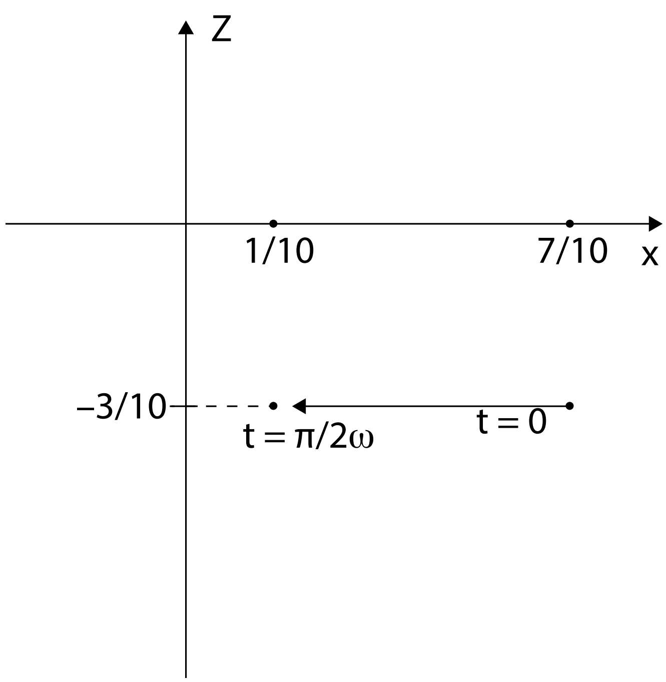
$$a_y=\frac{1}{2}Tr\left(\rho\sigma_y\right)=\frac{1}{10}\mathrm{Tr}\begin{pmatrix} 1 & 3\cos\omega t \\ 3\cos\omega t & 4 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}=\frac{1}{10}[3i\cos\omega t-3\cos\omega t]=0$$

$$a_z=\frac{1}{2}Tr\left(\rho\sigma_z\right).=\frac{1}{10}\mathrm{Tr}\begin{pmatrix} 1 & 3\cos\omega t \\ 3\cos\omega t & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}=\frac{1}{10}[1-4]=\frac{-3}{10}$$

Where is the vector $\vec{a}$ pointing at t = 0 and at t = π/2ω?

$at \quad t=0$	$t=\pi/2\omega$
$a_x(0)=\frac{7}{10}$	$a_x\left(\pi/2\omega\right)=\frac{1}{10}$
$a_y(0)=0$	$a_y\left(\pi/2\omega\right)=0$
$a_z(0)=-\frac{3}{10}$	$a_z\left(\pi/2\omega\right)=-3/10$

$\vec{a}$ is moving back and forth along a line in *xz* plane



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5.73 Quantum Mechanics I
Fall 2018

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