5.73

Quiz 20 ANSWERS

1. Write the 2×2 density matrix, $\rho(t)$, for the time evolving state that results from

$$|\psi(0)\rangle = 2^{-1/2}\begin{pmatrix}1\\1\end{pmatrix} = 2^{-1/2}\left[\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\right]$$

where $\mathbf{H} = \begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}$ and, for an eigenstate, $\Psi_j(t) = \psi_j e^{-iE_j t/\hbar}$.

* what is $|\Psi(t)\rangle$?
* what is $\rho(t)$?

$$|\Psi(t)\rangle = 2^{-1/2}\begin{pmatrix}1\\0\end{pmatrix}e^{-iE_1 t/\hbar} + 2^{-1/2}\begin{pmatrix}0\\1\end{pmatrix}e^{-iE_2 t/\hbar}$$

$$\rho(t) = \frac{1}{2}\begin{pmatrix}1 & e^{-i\omega_{12}t}\\ e^{i\omega_{12}t} & 1\end{pmatrix}$$

2. The detector is designed to see only $2^{-1/2}(|1\rangle - |2\rangle)$. Write the 2×2 $\mathbf{D}$ matrix.

$$D = \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1 & -1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

3. The $\mathbf{D}$ matrix is independent of time because the detector is not moving. But the expectation value for $\mathbf{D}$, the detected intensity, is time dependent because $\rho(t)$ contains time dependent coherence terms. Compute $\langle\mathbf{D}\rangle_t$ = Trace($\mathbf{D}\rho$).

$$\text{Trace}(D\rho) = \frac{1}{4}\left(D_{11}\rho_{11} + D_{12}\rho_{21} + D_{21}\rho_{12} + D_{22}\rho_{22}\right)$$

$$= \frac{1}{4}\left(1 + -e^{i\omega_{12}t} - e^{-i\omega_{12}t} + 1\right) = \frac{1}{2}\left(1 - \cos\omega_{12}t\right)$$

5.73 Quantum Mechanics I
Fall 2018