5.73

Quiz 13 ANSWERS

1.

$$\mathbf{a} = \begin{pmatrix} 0 & 1^{1/2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 2^{1/2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3^{1/2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \ddots & \ddots \\ 0 & 0 & 0 & 0 & 0 & \ddots \end{pmatrix} \qquad \mathbf{a}^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1^{1/2} & 0 & 0 & 0 & 0 \\ 0 & 2^{1/2} & 0 & 0 & 0 \\ 0 & 0 & 3^{1/2} & \ddots & 0 \\ 0 & 0 & 0 & \ddots & \ddots \\ 0 & 0 & 0 & 0 & \ddots \end{pmatrix}$$

A. Write the values of the following quantities:

$\mathbf{a}^\dagger_{2,3}$ $\boxed{0}$

$\mathbf{a}^\dagger_{5,4}$ $\boxed{\sqrt{5}}$

$(\mathbf{a}^\dagger\mathbf{a})_{6,7}$ $\boxed{0}$

$(\mathbf{a}^\dagger\mathbf{a})_{7,7}$ $\boxed{7}$

$[\mathbf{a},\mathbf{a}^\dagger]_{2,2}$ $\boxed{= (\mathbf{a}\mathbf{a}^\dagger)_{22} - (\mathbf{a}^\dagger\mathbf{a})_{22} = 3 - 2 = 1}$

$[\mathbf{a}^\dagger,\mathbf{a}]_{3,3}$ $\boxed{= -1}$

B. $\mathbf{a}^\dagger\mathbf{a}$ is called the "number operator", **N**. Why?

It reports the number of quanta in the basis state to which it is applied.

C. Selection rules are specified as the value of the quantum number on the left minus the value of the quantum number on the right. For the following products of six **a** or $\mathbf{a}^\dagger$ operators, what are the selection rules for nonzero matrix elements of:

$\mathbf{aaaa}^\dagger\mathbf{aa}$ $\Delta n =$ −4

$\mathbf{a}^\dagger\mathbf{a}^\dagger\mathbf{aaaa}^\dagger$ $\Delta n =$ 0

$\mathbf{aaa}^\dagger\mathbf{aa}^\dagger\mathbf{a}$? $\Delta n =$ −2

C. Which of the following matrices are Hermitian?

$\mathbf{a}$ no

$\mathbf{a}^\dagger$ no

$\mathbf{a} + \mathbf{a}^\dagger$ yes

$\mathbf{aa}^\dagger$ yes

$\mathbb{1}$ yes

D. Remember to start by first applying the operator on the far right: Evaluate

$(\mathbf{a}^\dagger)^7|n\rangle$ $=[(n+1)(n+2)(n+2)(n+4)(n+5)(n+6)(n+7)]^{1/2}\,|n+7\rangle$

$\langle n|\mathbf{a}^\dagger\mathbf{aaa}^\dagger\mathbf{a}^\dagger|n+1\rangle$. $= 0$