

5.73 Quantum Mechanics I
Fall, 2018

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Problem Set #8 (OPTIONAL)

Reading: Angular Momentum Handouts. C-TDL, pages

Problems: 643-676

1. CTDL, page 768, #6.
2. Consider the spectrum and dynamics of an alkali atom in electric and magnetic fields, both along the z-axis. You must consider three electronic states, the $n^2S_{1/2}$ ground state, the $n^2P_{1/2}$ and $n^2P_{3/2}$ excited states, and the $(n+1)^2S_{1/2}$ excited state. The Zeeman Hamiltonian is

$$H^Z = -\gamma B_z (\mathbf{L}_z + 2\mathbf{S}_z).$$

The Stark Hamiltonian is

$$H^S = \epsilon_z \mathbf{z}$$

where the only nonzero matrix elements of \mathbf{z} are $\Delta L = 1, \Delta M_L = 0$. For both Zeeman and Stark effects, you will want to work in the uncoupled basis set. For the Stark effect, the only relevant nonzero matrix elements are

$$\langle n^2P, M_L = 0, M_S | z | (n+1)^2S, M_L = 0, M_S \rangle \equiv \mu_1$$

where μ_1 is independent of M_S . The optical excitation Hamiltonian

$$H^{\text{opt}} = \bar{\epsilon}^{\text{opt}} \cdot (\hat{i}\mathbf{x} + \hat{j}\mathbf{y} + \hat{k}\mathbf{z})$$

has the same form as the Stark Hamiltonian, except that its nonzero elements are exclusively

$$\begin{aligned} \langle n^2S, M_L = 0, M_S | z | n^2P, M_L = 0, M_S \rangle &\equiv \mu_0 \\ \langle n^2S, M_L = 0, M_S | z^{-1/2} (x \pm iy) | n^2P, M_L = \mp 1, M_S \rangle &= 2^{1/2} \mu_0 \end{aligned}$$

Finally, there is the spin-orbit Hamiltonian,

$$H^{\text{SO}} = \zeta \mathbf{L} \cdot \mathbf{S}$$

which begs to be evaluated in the coupled representation. Let the energies of the relevant zero field states be:

$$\begin{aligned}E_{n^2S} &= 0 \text{ cm}^{-1} \\E_{(n+1)^2S} &= 20,000 \text{ cm}^{-1} \\E_{n^2P_{1/2}} &= 19,800 \text{ cm}^{-1} \\E_{n^2P_{3/2}} &= 19,900 \text{ cm}^{-1}.\end{aligned}$$

- A. At zero applied magnetic and electric fields, sketch the optical spectrum in the region between 19,700 and 20,100 cm^{-1} . Be sure to indicate the relative intensities of transitions if more than one resolvable feature occurs in the spectrum.
- B. At zero electric field, show how the z-polarized and x-polarized spectra evolve as the magnetic field is increased from 0 to a large enough value so that $|\gamma B_z| \gg hc(100 \text{ cm}^{-1})$. I am asking for two spectra.
- C. At zero magnetic field, show how the z and x polarized spectra evolve as the electric field is increased from 0 V/cm to a large enough value so that $\epsilon_z \mu_1$ is much larger than $hc(200 \text{ cm}^{-1})$.
- D. Now consider the nature of the quantum beats you might observe in the unresolved fluorescence following a 1 fs excitation pulse. Under what conditions would the observable quantum beats look like “population” beats (i.e., the z and x polarized fluorescence intensities would oscillate in-phase) and under what conditions would they look like polarization beats (i.e., the fluorescence intensity would appear to be rotating from z to x and back to z polarization)? This is an open-ended question. Be as explicit as you can about the various possibilities. These include either of two excitation polarizations, either of two detection polarizations, and the ($B_z \neq 0, \epsilon_z = 0$) and ($B_z = 0, \epsilon_z \neq 0$) magnetic and electric field possibilities.

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