9.07 INTRODUCTION TO STATISTICS FOR BRAIN AND COGNITIVE SCIENCES

Emery N. Brown

Lecture 10. Bayesian Analyses: Beta Probability Models

Waking Up from General Anesthesia

Analysis of Dendritic Spine Growth Data

Lecture 10 Bayesian Methods
Example 5.3: Analysis of Dendritic Spine Growth Data

259 interneurons recorded and 35 changed.

124 pyramidal neurons recorded and 0 changed.

Use a Bayesian analysis to determine if there is a different change probability for the pyramidal neurons compared with the interneurons.

Bayes' Theory

What is the best estimate of p given the observed data?

$$f(p \mid k) = \frac{f(p) f(k \mid p)}{f(k)}$$

Prior Probability Model

$$f(p_i) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} p_i^{\alpha-1}(1 - p_i)^{\beta-1},$$

f(p)BU (0,1); $\alpha = \beta = 1$

Binomial Probability Model for the Data

$$f(k_i|p_i) = \binom{n}{k_i} p_i^{k_i}(1 - p_i)^{n-k_i}.$$

Posterior Probability Model

$$f(p_i|k_i) = \frac{\Gamma(n + \alpha + \beta)}{\Gamma(k_i + \alpha)\Gamma(n - k_i + \beta)} \times p_i^{k_i+\alpha-1}(1 - p_i)^{n-k_i+\beta-1}.$$

Algorithm 10.1 (Bayesian Comparison)

$Sum = 0$

For $j = 1,\ldots,10,000$

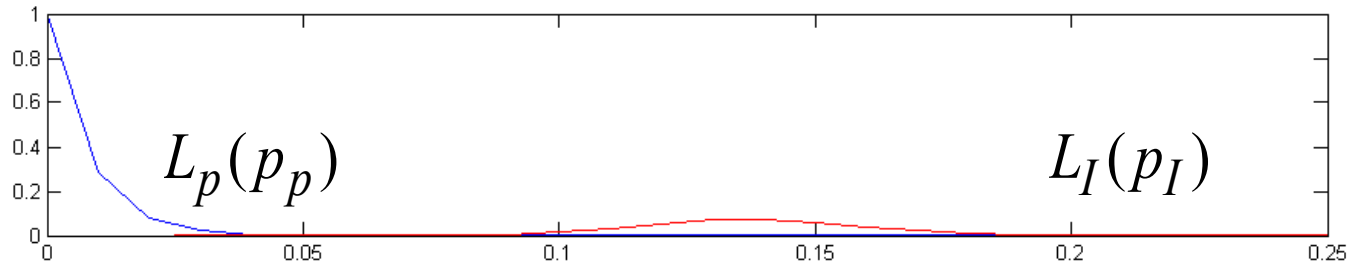
1. Draw $p_{p,j}$ from $f_p(p \mid k_p)$ and $p_{i,j}$ from $f_i(p \mid k_i)$
2. If $p_{i,j} > p_{p,j}$ $Sum \leftarrow Sum + 1$
3. If $j = 10,000$ then compute $\Pr(p_i > p_p) \doteq 10,000^{-1} Sum$

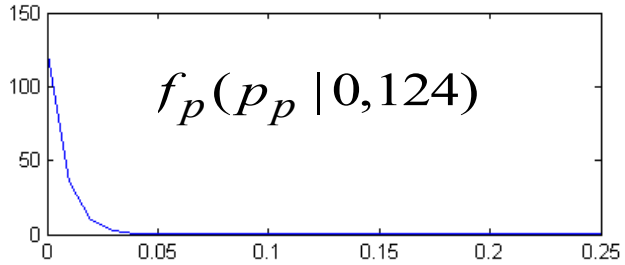
Make a histogram of $p_{i,j} - p_{p,j}$

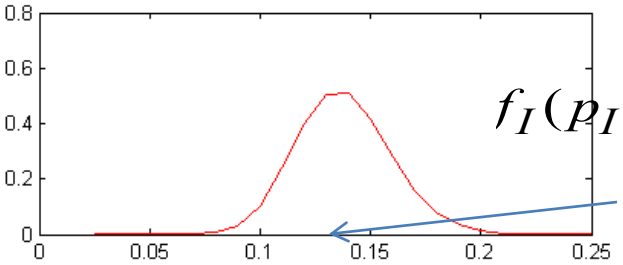
which is the probability density of the differences of the probabilities.

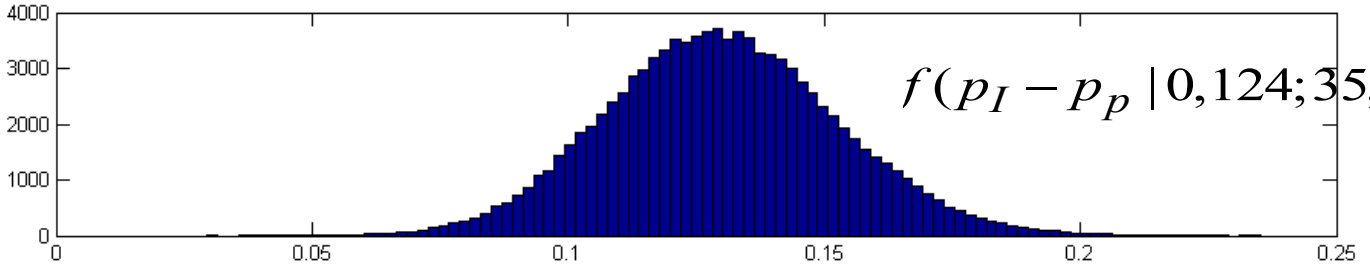
Lecture 10 Bayesian Methods

Example 5.3: Analysis of Dendritic Spine Growth Data









95% Credibility Interval : (0.0869 0.1760)

99% Credibility Interval: (0.0728 0.1919)

$$\Pr(p_I > p_p) = 0.9999$$

Reanimation from General Anesthesia by Administering Ritalin

Animals are anesthetized with propofol.

Group 1: Saline Group
0 of 6 animals have return of righting

Group 2: Ritalin Group
11 of 12 animals have return of righting

Are animals more likely to have return of the righting reflex after Ritalin than after saline?

Probability Model: Binomial

Is p in one group different from p in the other group?

Group 1: Binomial (n = 6, k = 0)

p = 0/6 = 0

Group 2: Binomial (n = 12, k = 11)

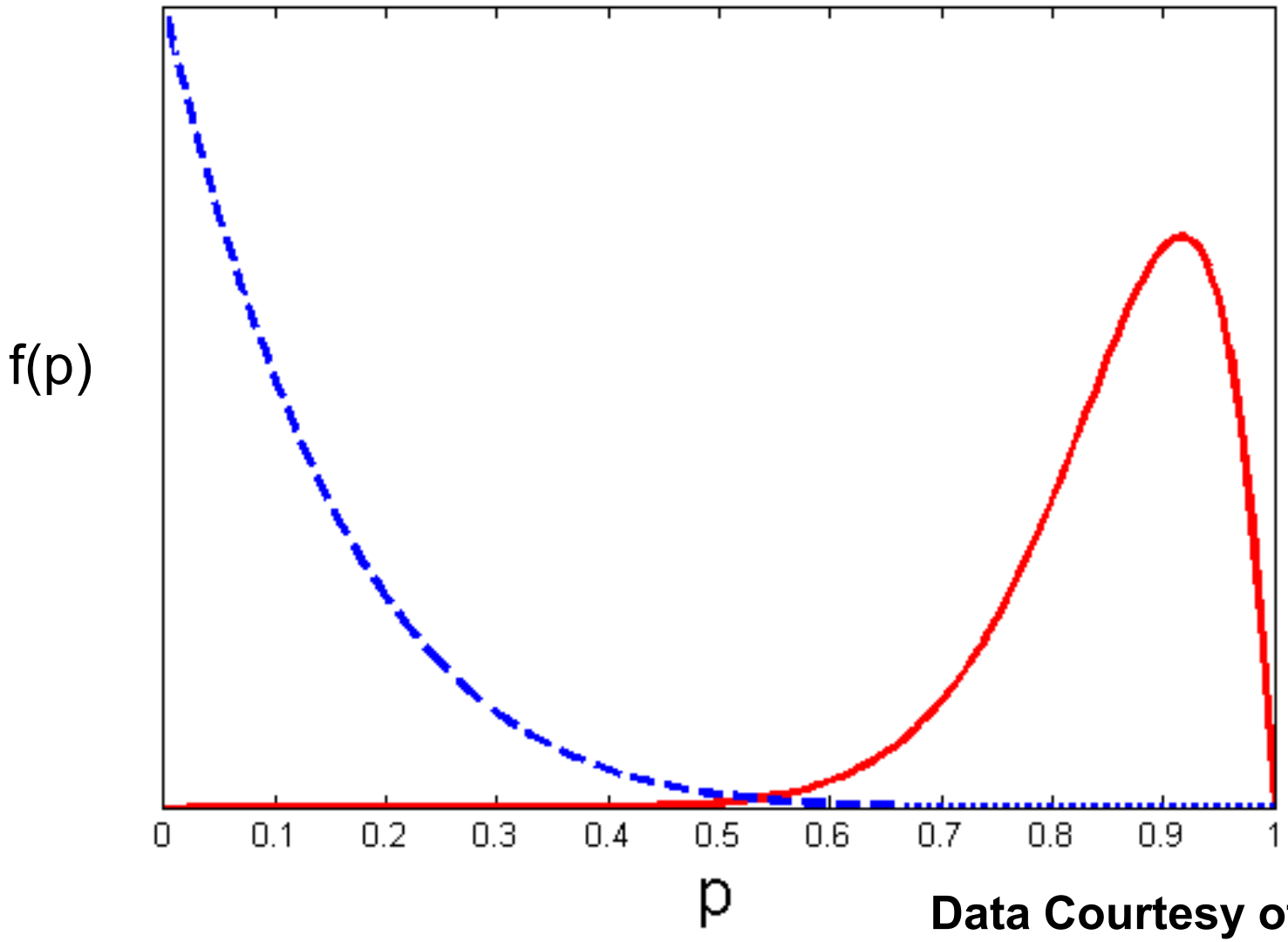
p = 11/12 = 0.92

Chemali et al. Anesthesiology 2012

Two Beta Posterior Densities

Saline — **Beta (alpha = 1, beta =7)**

Ritalin — **Beta (alpha = 12, beta = 2)**

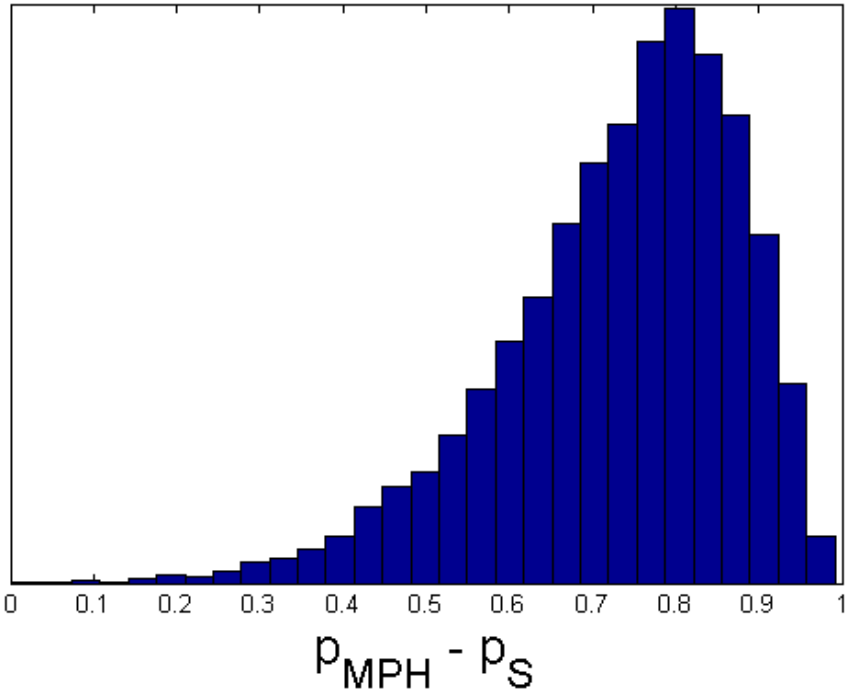


Chemali et al Anesthesiology 2012

Data Courtesy of Ken Solt
Analysis by Jessica Chemali

Probability Density of the Difference in the Probabilities

$f(p_{MPH} - p_S)$



$$\Pr(p_{MPH} > p_S) = \Pr(p_{MPH} - p_S > 0) > 0.95$$

Conclusion: There is greater than a 0.95 probability that the probability of an animal awakening after receiving Ritalin (MPH or methylphenidate) is greater than the probability of wakening after receiving the placebo.

9.07 Statistics for Brain and Cognitive Science
Fall 2016