

**20.430/6.561/10.539/2.795**  
**Fields, Forces, and Flows in Biological Systems**  
**Fall 2015**

**Problem Set # 4 (Electrical Sub-System)**

**Issued: Friday 10/9/15**

**Due: Monday, 5pm – 10/19/15**

**Reading Assignment:** Textbook, sections in **Chapter 2**: 2.1, 2.2; 2.3.1; 2.3.4; (skip 2.4), 2.5.1 (skip 2.5.2), 2.6, and 2.7.

(In general, we will not cover material on Magnetoquasistatics)

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Problem Sets should be turned in to the 20.430 FFF drop-off boxes, located to the right of the elevators on the 2<sup>nd</sup> floor of Building 16. **Please turn Problems 1 & 2 into Box 1, and Problem 3 into Box 2.**

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**Problem 1: From EM Waves to Quasistatics (a short derivation using scaling analysis)**

*Hint: Relates to Section 2.5.1, p48-49*

For slow enough time rates of change ( $\partial/\partial t \rightarrow 0$ ), we can neglect the  $(\partial\mu\mathbf{H}/\partial t)$  term in Faraday's law and arrive at the quasistatic form,  $\nabla\times\mathbf{E} \approx 0$ . As promised in lecture, the objective of this problem is to show that this quasistatic limit corresponds to the case where the wavelength  $\lambda$  of the EM is much longer than the characteristic length  $L$  of the system of interest (e.g., a tissue, cell, etc.). We use simplified dimensional analysis with Maxwell's equations as follows:

First, let's assume we're in free space ( $\rho_e = J = 0$ ). Gauss' Law:  $\nabla\cdot\epsilon\mathbf{E} = 0$  together with the quasistatic Faraday's law:  $\nabla\times\mathbf{E} \approx 0$  fully specifies the electric field  $\mathbf{E}_0$ .

(a) We now want to estimate the error we have made in calculating  $\mathbf{E}$  caused by neglecting the  $\partial\mu\mathbf{H}/\partial t$  term in Faraday's law. We can do this by first using Ampere's law (2.14 on page 36 of FFF) to find the  $\mathbf{H}_1(t)$  produced by the time varying zeroth order field  $\mathbf{E}_0(t)$ . Using dimensional analysis, with  $\nabla \rightarrow (1/L)$  and  $\partial/\partial t \rightarrow \omega$ , where  $L$  is the characteristic length and  $\omega$  is the frequency of the wave, find an analytical expression for  $\mathbf{H}_1$  in terms of  $L$ ,  $\omega$ ,  $\epsilon$ , and  $\mathbf{E}_0$ .

(b) Now plug your expression for  $\mathbf{H}_1$  back into the full Faraday's law  $\nabla\times\mathbf{E} = -\partial(\mu\mathbf{H}_1)/\partial t$  to find the error " $E_{error}$ " made by neglecting  $(\partial\mu\mathbf{H}/\partial t)$  in the first place. That is, find  $E_{error}$  in terms of  $\omega$ ,  $\mu$ ,  $\epsilon$ ,  $L$ , and  $\mathbf{E}_0$  (find the magnitude; forget the minus sign).

(c) From your answer to (b), form the ratio  $[E_{error}/E_0]$ . Show that when  $\lambda \gg L$ , the error is very small, and the quasistatic  $\mathbf{E}_0$  is sufficient to fully describe the electric field.

**Problem 2: Do Textbook Problem 2.3, parts (a) – (d)**

*Hint: Relates to Section 2.6.1 starting p51. In particular, Poisson's Eqn. (2.73) leads to part (b) Eqn. (2.128). Boundary conditions are on the electrical potential.*

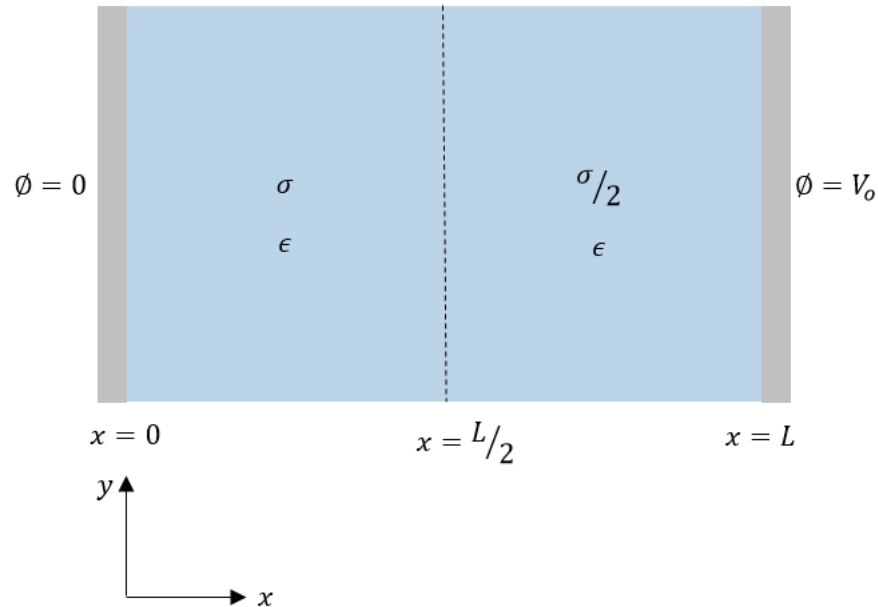
DfcV'Ya '&' " 'Z'ca 'hYI hVcc\_ 'fYa cj YX'Xi Y'hc'W'dmf][ \h'fYghf]W]cbg"

DfcV`Ya `&"" `Zfca `hYl hVcc\_`fYa cj YX`Xi Y`hc`Vtdnf][ \hfYghf]V]cbg"

### Problem 3: Gradient Gels for Protein Separation

Hint: Relates to Laplace's Eqn. (2.74) and boundary conditions that come from Eqns. (2.75), (2.76), (2.77), since you need boundary conditions on all surfaces surrounding the region(s) of interest (i.e.,  $x=0$ ,  $x=L/2$ ,  $x=L$ ,  $y=0$ ,  $y=d$ )

In the gel-based separation of proteins, a 'gradient' gel is often used. Such a gel is made by gradually changing the gel concentration, so that different sections of the gel have different pore sizes and conductivities. Consider the following system depicted below, where two different gel sections are made between two parallel electrodes.

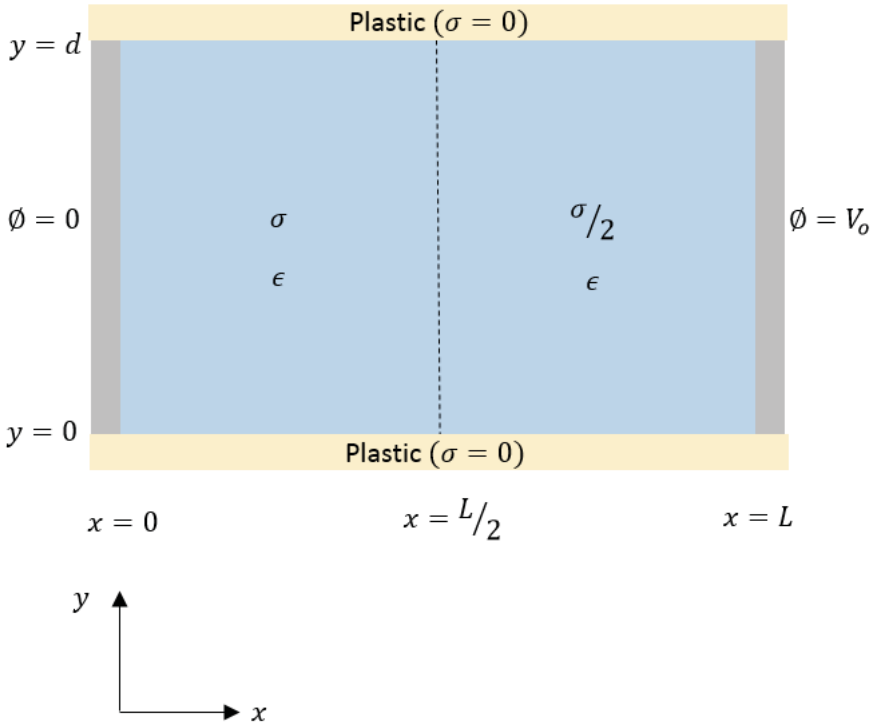


The electrode at  $x = 0$  is grounded, while the electrode at  $x = L$  is maintained at the fixed potential of  $V_0$ . The gel has is filled with a binary electrolyte solution that is half as concentrated on the right as on the left. Assume for now that the gel and the two electrodes are “infinitely long” in the  $y$  direction.

- State the appropriate boundary conditions for this system, and obtain an expression for the potential  $\Phi(x)$  in each compartment. Plot  $\Phi(x)$  versus  $x$  from  $x=0$  to  $x=L$  (please label the  $y$ -axis (i.e., the potential axis) appropriately). Hint: the solution for  $\Phi(x)$  in each compartment will contain two constants of integration, so you should have 4 clearly labeled boundary conditions.
- Obtain an expression for the electric field  $E(x)$ , and plot  $E(x)$  versus  $x$  from  $x=0$  to  $x=L$  (once again, please label the  $y$ -axis appropriately).
- Obtain an expression for the surface charge density  $\sigma_s$  at the internal interface  $x = L/2$ .

(d) Give a qualitative estimate of the approximate thickness of the surface charge formed at the internal gel interface (at  $x = L/2$ ).

(e) How would the situation change if the gel and electrodes were not infinitely long, but were instead bounded by insulating plastic plates ( $\sigma = 0$ ) at  $y=0$  and  $y=d$  as shown below? State the new appropriate boundary conditions at  $y = 0$  and  $y = d$ . Does this change your results for parts (b)-(d)?



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