MIT 16.90 Spring 2014: Problem Set 3

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Due: Monday March 3, in class

Problem 2. 1 *Reading Assignment*

- 2.1. Overview
- 2.2. Partial Differential Equations.
- 2.3. Introduction to Finite Difference Methods.

Problem 2. 2 *Stiffness of a flight simulator*

Consider the governing equations of our flight simulator

$$\frac{dv}{dt} = \frac{-D - m\,g\sin(\theta)}{m}$$
$$\frac{d\theta}{dt} = \frac{L - m\,g\cos(\theta)}{m\,v}$$
$$\frac{d\alpha}{dt} = \frac{inp - \alpha}{\tau}$$
$$\frac{dh}{dt} = \sin(\theta)v;$$

where inp is the yoke input which we assume is a constant for now. $\tau = 0.01$ is the timescale of the static longitudinal stability. Also,

$$D = \frac{1}{2}\rho v^2 A\,C_D$$
$$L = \frac{1}{2}\rho v^2 A\,C_L$$
$$C_L = 2\pi\alpha$$

The air density $\rho = 1kg/m^3$. The drag coefficient $C_D = .025$. The wing area $A = 16m^2$. The airplane mass $m = 1000kg$. The gravity constant $g = 9.8$.

Linearize the ODE at $v = 100m/s, \theta = \alpha = h = 0$. (Hint: plug both (v, θ, α, h) and $(v + \delta v, \theta + \delta\theta, \alpha + \delta\alpha, h + \delta h)$ into the equations. Remember not to neglect $\delta C_L, \delta D$ and δL.) Write the resulting linear ODE for $(\delta v, \delta\theta, \delta\alpha, \delta h)$ in matrix form. Is the ODE stiff? Why or why not?
