

MIT 16.90 Spring 2014: Problem Set 3

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Due: Monday March 3, in class

Problem 2.1 *Reading Assignment*

- 2.1. Overview
 - 2.2. Partial Differential Equations.
 - 2.3. Introduction to Finite Difference Methods.
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Problem 2.2 *Stiffness of a flight simulator*

Consider the governing equations of our flight simulator

$$\begin{aligned}\frac{dv}{dt} &= \frac{-D - mg \sin(\theta)}{m} \\ \frac{d\theta}{dt} &= \frac{L - mg \cos(\theta)}{mv} \\ \frac{d\alpha}{dt} &= \frac{inp - \alpha}{\tau} \\ \frac{dh}{dt} &= \sin(\theta)v;\end{aligned}$$

where inp is the yoke input which we assume is a constant for now. $\tau = 0.01$ is the timescale of the static longitudinal stability. Also,

$$\begin{aligned}D &= \frac{1}{2}\rho v^2 A C_D \\ L &= \frac{1}{2}\rho v^2 A C_L \\ C_L &= 2\pi\alpha\end{aligned}$$

The air density $\rho = 1\text{kg}/\text{m}^3$. The drag coefficient $C_D = .025$. The wing area $A = 16\text{m}^2$. The airplane mass $m = 1000\text{kg}$. The gravity constant $g = 9.8$.

Linearize the ODE at $v = 100\text{m}/\text{s}, \theta = \alpha = h = 0$. (Hint: plug both (v, θ, α, h) and $(v + \delta v, \theta + \delta\theta, \alpha + \delta\alpha, h + \delta h)$ into the equations. Remember not to neglect $\delta C_L, \delta D$ and δL .) Write the resulting linear ODE for $(\delta v, \delta\theta, \delta\alpha, \delta h)$ in matrix form. Is the ODE stiff? Why or why not?

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