# Electromagnetic Formation Flight

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# INTRODUCTION

## Description of the Effort

The Massachusetts Institute of Technology Space Systems Lab (MIT SSL) and the Lockheed Martin Advanced Technology Center (ATC) are collaborating to explore the potential for an Electro-Magnetic Formation Flight (EMFF) system applicable to Earth-orbiting satellites flying in close formation.

## Progress Overview

At MIT, work on EMFF has been pursued on two fronts: the MIT conceive, design, implement and operate (CDIO) class, and the MIT SSL research group.

Recent work in the MIT CDIO class includes the finalizing of the design for a planar EMFF test bed. The class will be holding its Critical Design Review (CDR) on December 5, 2002, and the final hardware design presented in their CDR will be described in the December progress report.

Recent work in the MIT SSL has focused on investigating specific applications of the EMFF in Low Earth Orbit (LEO), particularly with regards to managing angular momentum accumulation in LEO due to periodic disturbances. **The following report discusses in detail the use of EMFF to manage angular momentum buildup,** specifically due to the Earth's magnetic field and due to its oblateness about the equator, also known as the $J_2$ perturbation. Results of these analyses indicate that EMFF could serve well in this application, replacing the use of thrusters to correct for $J_2$ accelerations, and thus removing an array's dependence on non-renewable fuel supplies. These results are promising, as they suggest yet another application in which EMFF may be quite useful.

# ANGULAR MOMENTUM MANAGEMENT

## 1.  Overview

EMFF has the property that all the forces generated by the electromagnets are internal to the system.  Excluding any disturbance interactions with the Earth's magnetic field, EMFF cannot apply any external forces to the formation. This prevents EMFF from having the ability to change the *inertial* position of a formation.  It also prevents changes in the total system angular momentum.  While reaction wheels are used to temporarily store angular momentum changes in a formation, they are not able to change the *total* system angular momentum.

In an ideal situation, this would not be an issue, since the angular momentum of the system would remain constant. However, disturbance forces cause unwanted torques on a satellite formation and cause a net gain in its system angular momentum.  If left unchecked, the reaction wheels will eventually saturate and become useless.  These disturbance torques must be accounted for, and methods for removing this unwanted angular momentum are suggested in this report.

## 2.  Earth's Magnetic Field

One disturbance source that is immediately apparent is the Earth's magnetic field. Whenever any of the EMFF electromagnets are energized, they interact with the Earth's magnetic field.  This disturbance will manifest itself as disturbance forces and torques on the satellite formation.  These disturbance forces and torques will cause a net gain in the angular momentum of the satellite formation.

In order to characterize this effect, the Earth can be thought of as one large electromagnetic dipole.  This is the same simplification technique used in previous studies, where interactions (forces and torques) between the electromagnets on different spacecraft are modeled as functions of the product of the magnetic dipole strengths.  One novel way of canceling the disturbance torques and forces due to the Earth's magnetic field is to reverse the polarity of every spacecraft dipole at the same time [Hashimoto et al. 2002]. Since the electromagnetic forces are functions of the product of the dipoles, the inter-spacecraft forces are unchanged.  However the disturbance forces and torques due to the Earth's dipole have switched sign, thus canceling out the disturbance forces.  This polarity switching could be done at a low frequency; the dipoles would switch only when the accumulated angular momentum becomes large, or the switching could also be done at a relatively high frequency, essentially preventing any angular momentum build-up.

Since we have the ability to apply a net torque to the satellite formation in one direction or the other, the Earth's magnetic field could be utilized to remove the overall net angular momentum of the system. If the system were gaining angular momentum due to some other disturbance torque, the Earth's magnetic field could be used to reduce the system angular momentum.  Also, since the inter-spacecraft forces are once again products of the magnetic dipole strengths, we can selectively turn up one satellite's dipole strength and reduce the other spacecraft's dipole strength.  The inter-satellite forces and torques would remain unchanged, but

the magnetic disturbance force would be focused on one specific satellite. *Thus individual satellites could be targeted for momentum exchange*.

### 3. LEO and the $J_2$ Disturbance

Satellite formations in Low Earth Orbit (LEO) are subjected to variations in the Earth's gravitational potential. The largest variation is known as the $J_2$ gravitational potential constant. The $J_2$ term refers to the fact that the Earth is not round, but has a mass bulge around the equator. This mass bulge has three main effects on the satellite formation: it changes the orbital period of the satellite, it causes the formation to separate in the cross-track direction, and it causes the formation as a whole to rotate about its own angular momentum vector.

One effect of the $J_2$ geopotential is that it causes a satellite's orbital angular momentum vector to precess about the vector coincident with the North Pole. The rate of this precession is dependent primarily on the satellite inclination and altitude. Except for a couple of specific geometry formations, satellite formations usually have satellites with slightly different inclinations and altitudes and thus different rates of precession of the orbital angular momentum vector. The different rates of precession cause the formation to drift apart, since their orbital planes are not aligned anymore. *Internal forces can be used to counteract these differential $J_2$ forces*.

In order to counteract these disturbance forces, shear forces must be used, and there is a net gain or loss in the formation angular momentum. The loss or gain of the angular momentum is dependent, not on the direction of the EMFF dipole as with the Earth's magnetic field, but on the geometry of the formation. Certain satellite formation shapes and phasings produce a net gain in the formation angular momentum and other formations produce a net loss. Once again, if this angular momentum gain or loss is left unchecked, the reaction wheels will become saturated as they absorb more and more of the angular momentum.

Because of the ability to select the net gain or loss of the angular momentum vector by simply re-phasing the formation, angular momentum change due to $J_2$ disturbances does not prevent EMFF from being a viable option for satellites within LEO. *In fact, the $J_2$ disturbance, much like the Earth's magnetic field, can be used to add or lose angular momentum from the system that is gained from other sources*. Overall, torque management is key to the success of EMFF. Luckily there are ways to change the system angular moment buildup without the use of thrusters.

### 4. Angular Momentum Management in the Presence of J2 Disturbances

The $J_2$ effect of the Earth's geopotential is a disturbance acceleration on satellite formations that must be counteracted. This section looks in detail at the effects of fighting the separation of an array in the cross-track direction.

The acceleration of a satellite due to the $J_2$ force is defined as:

$$\vec{J}_2(\vec{r}) = -\frac{3}{2}\frac{J_2 \mu R_e^2}{r^4}[(1-3\sin^2 i \sin^2 \theta)\hat{x} + (2\sin^2 i \sin\theta\cos\theta)\hat{y} + (2\sin i \cos i \sin\theta)\hat{z}) \qquad (1)$$

where $\vec{r}$ is the position vector of the satellite, and $\hat{x}-\hat{y}-\hat{z}$ is a local vertical, local horizontal (LVLH) coordinate system with the origin of the coordinate system coinciding with a reference orbit. The $\hat{x}$ vector points in the radial direction, the $\hat{z}$ vector is perpendicular to the orbital plane and points in the direction of the angular momentum vector. Finally, the $\hat{y}$ vector completes the orthogonal triad, pointing in the direction of movement. The $\hat{x}-\hat{y}-\hat{z}$ coordinate system is a curvilinear coordinate system. The $\hat{x}$ vector remains unchanged, while the $\hat{y}$ and $\hat{z}$ vectors curve along a sphere with radius $r_{ref}$.

While $J_2$ disturbances acting on the formation as a whole are important, it is the differential disturbance accelerations between the satellites in the cluster that causes the formation to separate.

Linearizing the $J_2$ acceleration about one of the satellites produces the following result:

$$\nabla \bar{J}_2(r,\theta,i) =$$

$$\frac{6\mu J_2 R_E^2}{r_{ref}^5}
\begin{bmatrix}
(1-3Sin^2i\,Sin^2\theta) & Sin^2i\,Sin\,2\theta & Sin\,2i\,Sin\,\theta \\
Sin^2i\,Sin\,2\theta & -\frac{1}{4}-Sin^2i\,(\frac{1}{2}-\frac{7}{4}Sin^2\theta) & -\frac{Sin\,2i\,Cos\,\theta}{4} \\
Sin\,2i\,Sin\,\theta & -\frac{Sin\,2i\,Cos\,\theta}{4} & -\frac{3}{4}+Sin^2i\,(\frac{1}{2}+\frac{5}{4}Sin^2\theta)
\end{bmatrix}
\qquad (2)$$

Assuming a two-satellite formation with one satellite at the center of a closed Hills orbit, the second satellite will orbit the first (assuming no disturbances) as defined by Hill's equations:

$$\vec{s} = \begin{bmatrix}
x = a\cos(\theta+\lambda_1) \\
y = \frac{a}{2}\sin(\theta+\lambda_1) \\
z = b\cos(\theta+\lambda_2)
\end{bmatrix} \qquad (3)$$

One approximation to calculating the differential forces over one orbital period is to take the non-perturbed motion and dot it with the gradient of the $J_2$ disturbance:

$$F_{differential} = \vec{s}.\nabla \bar{J}_2 \qquad (4)$$

where $\vec{s}$ is the relative position of one satellite with respect to another. This differential force is then converted to axial and shear components. The axial force needed to counteract the differential $J_2$ forces does not produce any torque on the satellite formation. It is the shear component of the force that produces torque on the satellite formation and must be counteracted. Forces in the radial direction are along the axis between the two satellites, and the tangential and normal accelerations are the shear forces with the tangential force lying in the relative orbital

plane, and the normal force, normal to this plane. Forces in the tangential and normal direction produce a torque on the satellite system and the net angular momentum gain or loss must be accounted for.

Figure 1 shows the disturbance accelerations due to the $J_2$ geopotential on a 50m satellite formation. The reaction wheel will absorb the fluxuation over an orbital period, but if the accelerations in the normal and tangential direction integrate to a non-zero number, the reaction wheels will eventually saturate.
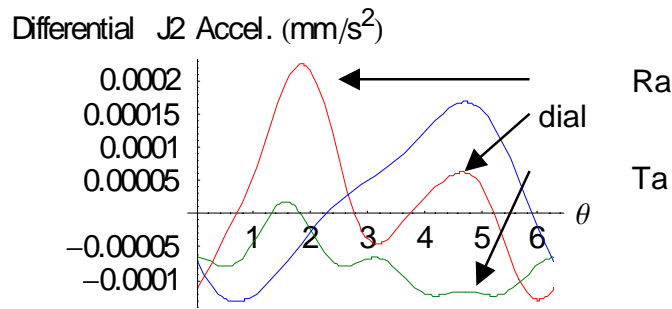


**Figure 1: Differential J$_2$ Accelerations over one orbit**
**(50m Separation Distance)**

Looking at equation 4, the relative obrit between the two satellites can be changed by varying the following parameters $(a,b,\lambda_1,\lambda_2)$. The variables a and b just change the size and shape of the formation, while $\lambda_1$, $\lambda_2$ control the phasing of the formation. The phasing plays a very important roll in the direction of the gain. Figure 2 shows the integrated value of the differential acceleration over one orbit as a function of $\lambda_1$, $\lambda_2$. There are two things that should be taken from these plots. First, certain phasing angles produce zero net gain in the torque, both in the normal and tangential directions. These formation do not have a net gain in the system angular momentum when counteracting the differential $J_2$ disturbances. Also, the net gain appears to be somewhat sinusoidal, and so the net gain is either positive or negative, depending on the phasing angles.
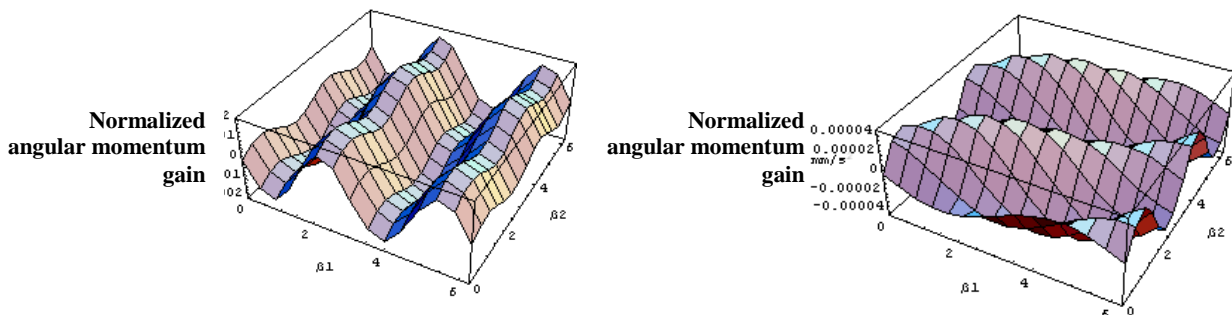


**Figure 2: Net Angular Momentum Gain Over One Orbit**
**(Left) Tangential Direction, (Right) Normal Direction**

Overall, Figure 2 shows that while for most satellite formations, there is a net gain in the system angular momentum when counteracting the $J_2$ disturbance, *the direction of the gain can be chosen at will by simply rephasing the satellite formation, and thus can be used to control the system angular momentum.*