

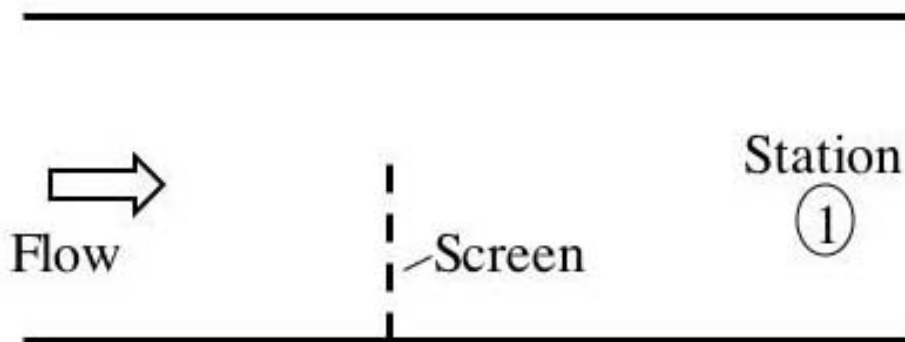
Practice Concept Quiz #2

1. [Used as Concept Quiz #3 (2004)] We have seen that whether a flow is rotational or not can have a major difference on its behavior. Below are some examples of situations encountered in fluids engineering. State whether the flow at the downstream station, station 1, *is* or *is not* rotational. If it is rotational, indicate the sense of the vorticity. **As previously, one to several sentences explaining your reasoning, bolstered by equations and/or a sketch if you think appropriate, are needed for full credit.**

You can assume that the flow *at the station* can be approximated as inviscid, i.e. that the effect of wall boundary layers on the overall velocity distribution does not need to be included in your response.

a) Fluid flows through a two-dimensional bent duct. The velocity at a station *midway through the bend* is uniform across the duct. The Mach number is everywhere small compared to unity. Station 1 is in a straight duct downstream of the bend.

b) Fluid flows through a screen that covers half the area in a constant area duct, as in the figure below. The screen is normal to the duct wall. The Mach number is everywhere small compared to unity.

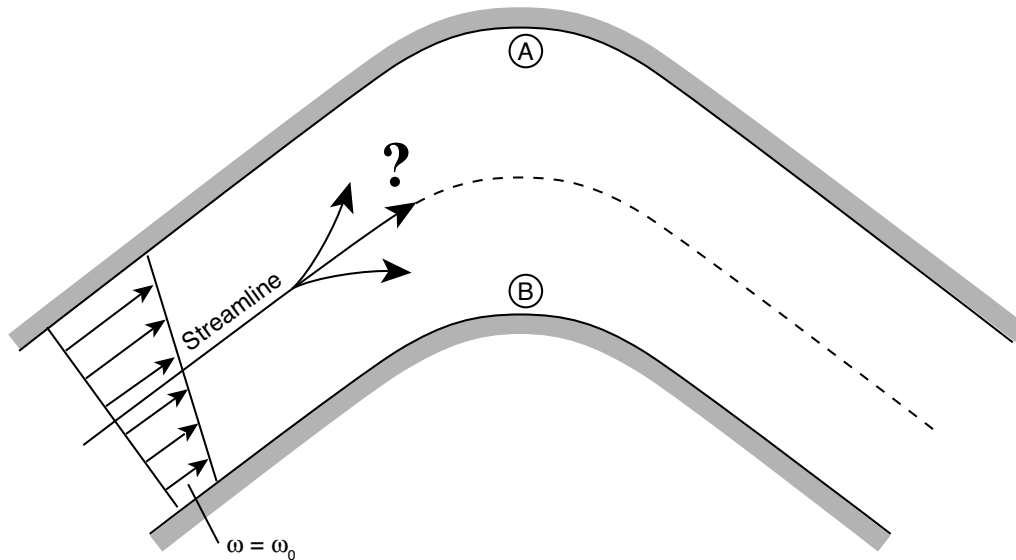


c) Heat is added to the *upper half of the stream* in a constant area, two-dimensional, duct. The temperature of the heated fluid that exits the region of heat addition is ten times the (uniform) upstream temperature. The Mach number is everywhere small compared to unity. Station 1 is downstream of the region of heat addition.

d) Fluid flows through a straight oblique shock that spans across an inlet. The flow upstream of the shock is uniform. Station 1 is downstream of the shock.

e) A compressor stage (a rotor and a stator) has non-uniform work addition, with the work added to the flow increasing from the hub to the tip (i.e., from the inner boundary of the annulus to the outer casing). The flow entering the stage is uniform and axial and the flow leaving the stage is axial. The flow through the stage can be considered isentropic and the Mach numbers are everywhere small compared to unity. Station 1 is downstream of the stage.

2. An inviscid constant density fluid flows through a bend. The flow is two-dimensional with *uniform vorticity*, ω_0 . Far upstream and downstream of the bend the pressure is uniform. Stations A and B are on the outer wall and the inner wall of the bend respectively.



- Is p_A less than, greater than, or equal to p_B ? Why? (A sentence or two is needed to explain the reason for the answer. You can use equations if they help the explanation. No credit without the explanation.)
- Is u_A less than, greater than, or equal to u_B ? Why? (A sentence or two is needed to explain the reason for the answer. You can use equations if they help the explanation. No credit without the explanation). Are there situations which might have different answers? What would be a metric to define what these situations are?
- A streamline at the middle of the upstream straight section of the channel is indicated. As the streamline nears the bend will it continue in the middle of the channel, will it deflect inwards, or will it deflect outwards. Why? (A sentence or two is needed to explain the reason for the answer. You can use equations if they help the explanation. No credit without the explanation.)

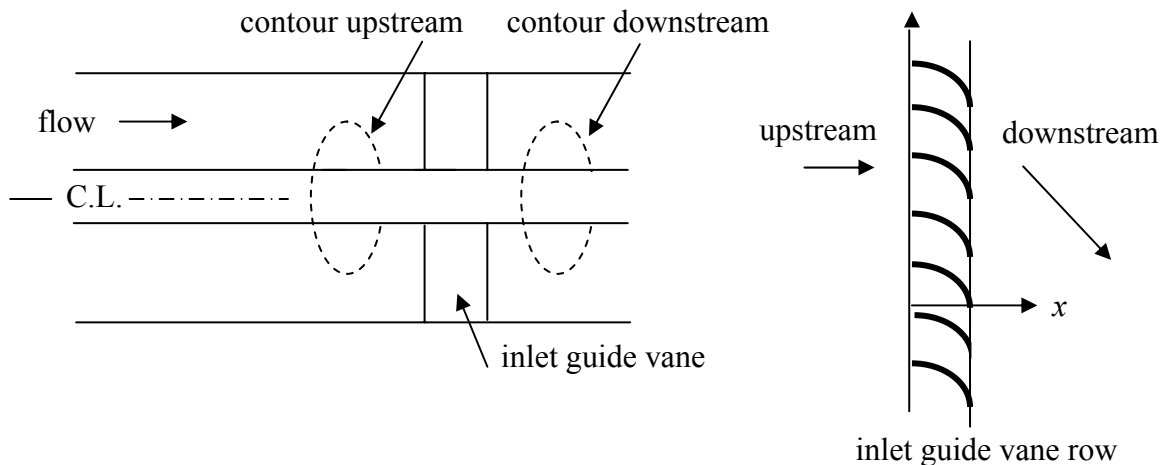
4. An inviscid fluid flows in an annular region in a turbomachine, through a row of inlet guide vanes, which turn the flow to some angle (say 30 degrees). Upstream of the vane row the flow in the annulus is axial, and a circular contour has no circulation (see figure). Downstream of the row, the flow has swirl, and a circular contour at the same radius has a non-zero circulation.

a) How is Kelvin's theorem related to this observation? (What does Kelvin's theorem say about the evolution of the upstream contour?)

b) What relation does the circulation around the vanes have to the circulation around the contour? Why?

c) If there is circulation round the contour, that must mean that vortex lines are enclosed. Where are they? Draw the vortex lines.

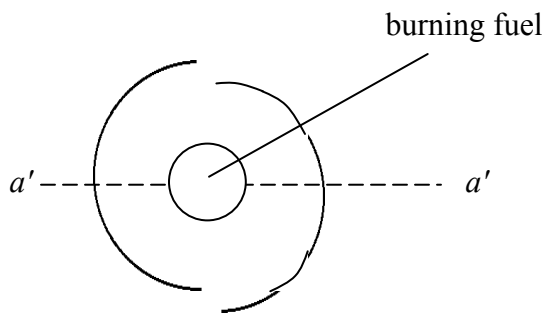
d) Suppose the fluid were viscous. Would the flow be much changed? Would your picture of the vortex lines be much different? Would any of the arguments used be much changed? Why or why not?



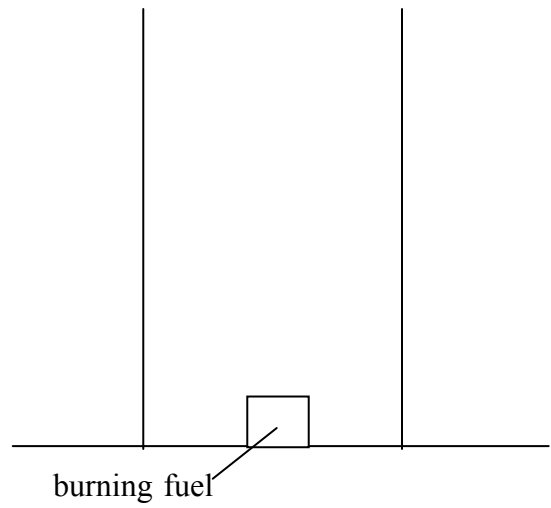
A. Annulus and inlet guide vane showing contours

B. "Unrolled" turbomachine inlet guide vane row

5. Can we make a "fire whirl" by placing two offset half cylinders around a can of burning fuel (see below). Why or why not?

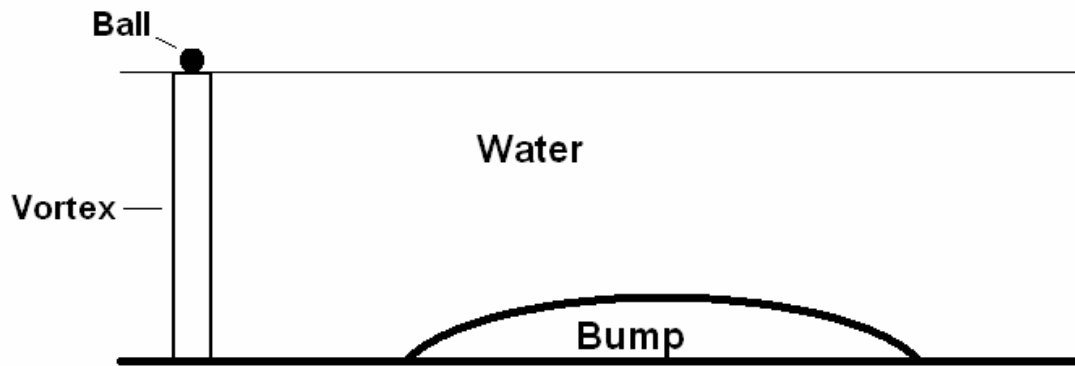


Top view



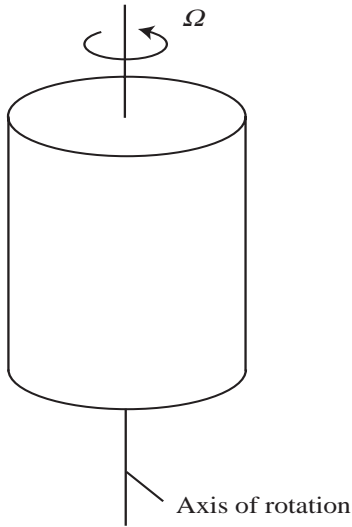
What would the vortex lines look like?

6. The following question is based on an experiment performed by former Institute Professor A. H. Shapiro and described in his film, *Vorticity*. A sketch of the water channel experiment is shown below. A vertical vortex tube is generated at the left end of the channel and travels towards the right end of the channel. Initially, a small, light ball floats over the center of the vortex.



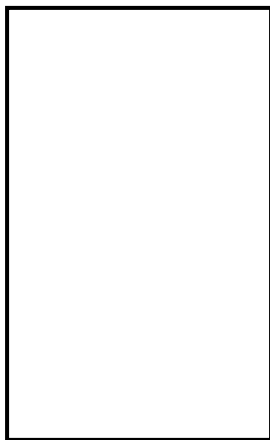
What happens to the rotational speed of the ball as: (a) it floats over the bump, and (b) as it moves downstream of the bump?

7. [Concept Quiz #2 (2004)] A viscous constant density fluid completely fills a cylindrical container, which is rotating around an axis through the center ($r = 0$) of the cylinder with angular velocity Ω . The rotation can be considered to have been going on for a very long time.

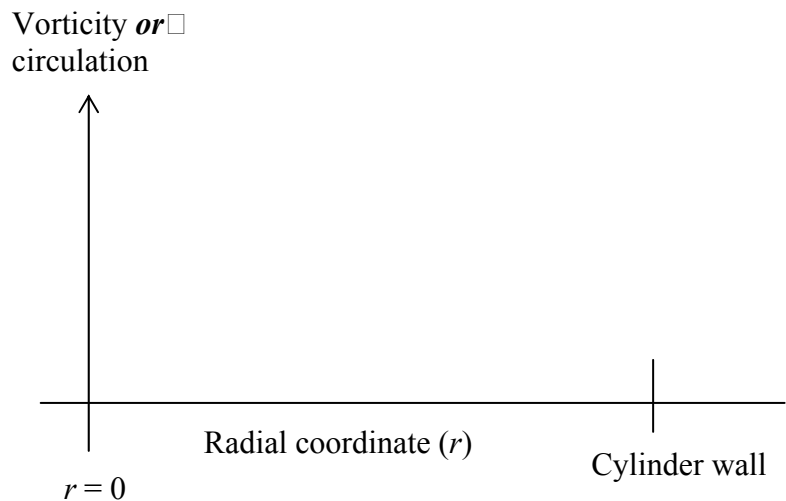


- Sketch the *vortex lines* in the container (use the diagram on the left in the figure below).
- Sketch the *vorticity distribution at mid-height* as a function of radius (use the axes below).
- Sketch the *circulation distribution at mid-height* as a function of radius (use the axes below).

In this, as in the questions below, a sentence or two about *why* each of your sketches look the way they do is expected for full credit.

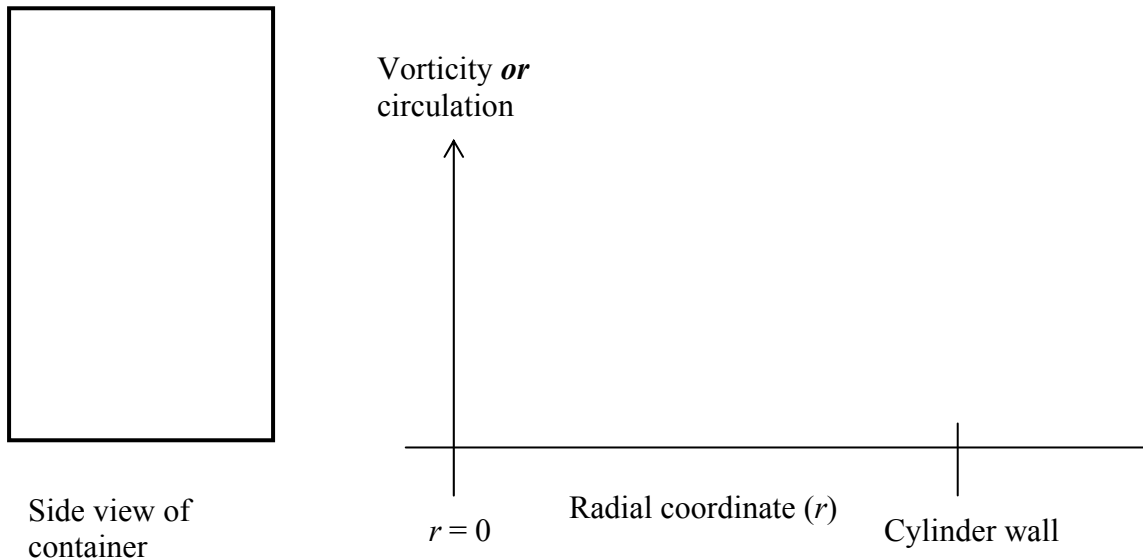


Side view of container

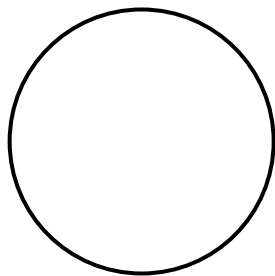


The cylindrical container is instantaneously *brought to rest* (side walls, top, and bottom). For the fluid in the container *a short time* after the container is stopped from rotating:

- d) Sketch the *vortex lines* in the container (use the diagram on the left on the figure below).
- e) Sketch the *vorticity distribution at mid-height* as a function of radius (use the axes below).
- f) Sketch the *circulation distribution at mid-height* as a function of radius (use the axes below).



- d) Will the fluid *near the bottom of the container* start to move inward, outward, or stay at the same location when the container is stopped from rotating? Why? Draw what you think will be the streamlines near the bottom of the container on the diagram below?

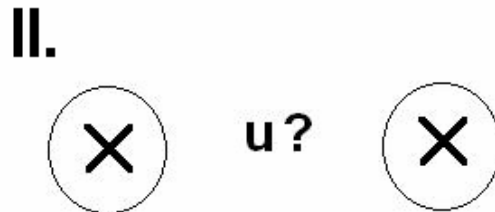


Top view

8. A - A top-down view of two infinite vortex tubes is shown in Figure I. Assuming identical vortex tube strengths, describe the velocity of fluid, u , in the region between the vortex tubes. Also, describe the motion of the vortex tubes.



B – Answer the above questions for Figure II.



C – What would be the situation if the two vortices were NOT of equal strength in part B, say if one were twice the strength of the other. (This question suggested by Professor Jun Li)

9. Based on your knowledge of the flow field for two vortices of opposite sense (or any other considerations you wish to bring to bear), can you describe the two-dimensional flow field associated with a line vortex (i.e., an infinite, straight, two-dimensional vortex line) of strength Γ , located a distance h above a plane surface? The fluid is incompressible and inviscid and extends everywhere above the plane wall.

10. (This question was also suggested by Professor Li.) The question is based on an experiment carried out by Professor Shapiro in a large diameter (six feet) water tank, also described in Shapiro's film on vorticity. The experiment addresses the fact that the earth is rotating and is not an inertial system, even though we typically assume it is when considering fluid machinery and propulsion. Shapiro's experiment was designed to amplify, through an enormous shrinkage of the diameter of a fluid contour as it moved towards the drain hole in the center of the tank floor, the angular velocity associated with the earth's rotation. Professor Li's question refers to the lift fan on the JSF (F-35) aircraft, which is horizontal with its inlet on top of the aircraft. The fan diameter can be taken to be, for purposes of this problem, roughly a meter. At low vertical velocities, no wind, and no forward speed the capture stream tube area, in other words the area of the stream tube that enters the fan, would be many times the fan area. In that situation could the effect of the earth's rotation ever be appreciable in determining fan performance? Can we answer "yes" or "no" from what we know about vorticity and circulation?