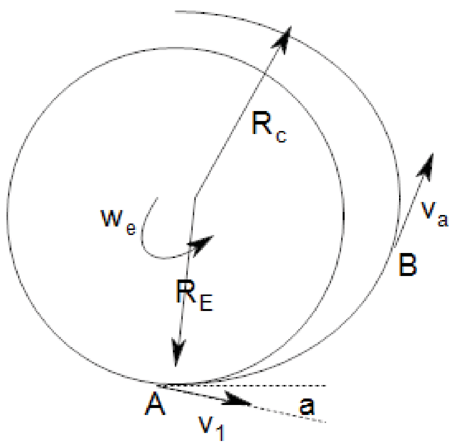
Homework 1: Preliminary Design of a Satellite Launcher

a) Velocity Calculations:



Point A: Start of ascent trajectory
Point B: Apogee of ascent trajectory

Conservation of Angular Momentum:

$$v_1 \cos\alpha \, R_E = v_a R_c$$

Rearranging becomes:

$$v_a = v_1 \frac{R_E}{R_c} \cos\alpha \quad (1)$$

Conservation of Energy:

$$\frac{v_1^2}{2} - \frac{\mu_E}{R_E} = \frac{v_a^2}{2} - \frac{\mu_E}{R_E} = \frac{v_1^2}{2}\left(\frac{R_E}{R_c}\cos\alpha\right)^2 - \frac{\mu_E}{R_E}$$

$$\frac{1}{2} v_1^2 \left(1 - \frac{R_E^2}{R_c^2}\cos^2\alpha\right) = \frac{\mu_E}{R_E} - \frac{\mu_E}{R_c}$$

$$v_1 = \sqrt{\frac{2\mu_E}{R_E}\left(\frac{1-\frac{R_E}{R_C}}{1-\frac{R_E^2}{R_C^2}\cos^2\alpha}\right)} \quad (2)$$

Values, Constants, and Given Parameters:

$R_E = 6370\ km$
$R_C = 6870\ km$
$\mu_E = 3.98e14\ \frac{m^3}{s^3}$
$\alpha = 20°$

Substituting Values:

Using equation (2) to find v_1:

$$v_1 = \sqrt{\left(\frac{2*3.98e14}{6.37e5}\right)\left(\frac{1-637/687}{1-(637/687\cos 20°)^2}\right)}$$

$$v_1 = 6145\,\frac{m}{s}$$

Using equation (1) to find v_a:

$$v_a = 6145 \times \frac{637}{687}\cos 20°$$

$$v_a = 5354\,\frac{m}{s}$$

Orbital Velocity:

$$v_c = \sqrt{\frac{\mu_E}{R_C}} \quad (3)$$

$$v_c = \sqrt{\frac{3.98e14}{6.87e6}}$$

$$v_c = 7611\,\frac{m}{s}$$

b) Stage ΔV Calculations:

We allocate the full "apogee kick" to the third stage:

$\Delta V_3 = v_c - v_a$ (4)

$\Delta V_3 = 7611\frac{m}{s} - 5354\frac{m}{s}$

$\Delta V_3 = 2257\frac{m}{s}$

The initial ascent velocity v_1 contains the velocity increments of the 1st and 2nd stages, plus the contribution from Earth rotation, minus the losses from gravity and drag:

$v_1 = \Delta V_1 + \Delta V_2 + \omega_E R_E \cos\alpha - \Delta V_G - \Delta V_D$ (5)

By design:

$\Delta V_1 = \Delta V_2 = \frac{1}{2}(v_1 - \omega_E R_E \cos\alpha + \Delta V_G + \Delta V_D)$

To solve:

$\omega_E R_E \cos\alpha = 7.268e(-5)\frac{rad}{s} \times 6.37e6\ m \times \cos 20°$

$\omega_E R_E \cos\alpha = 435\frac{m}{s}$

To estimate the gravity loss (1st stage only) we <u>assume</u>:

$\sin\gamma = 1 - \frac{t}{t_{b1}}$ (6)

($\sin\gamma$ is linear between $\gamma = 1$ at $t = 0$ and $\gamma = 0$ at $t = t_{b1}$)

Determining relations among parameters:

$t_{b1} = \frac{M_{p1}}{\dot{m}_1} = \frac{M_{p1}}{\left(F_1/c\right)} = \frac{M_{p1}}{M_{01}*3g}c$

$\frac{M_{p1}}{M_{01}} = 1 - \frac{M_{f1}}{M_{01}} = 1 - e^{-\frac{\Delta V_1}{c}}$

We need to make a preliminary guess at ΔV_1. Take for now $\Delta V_1 = 3200\frac{m}{s}$.

$\frac{M_{p1}}{M_{01}} = 1 - e^{\frac{-3200}{9.8*270}} = 0.7016$

$t_{b1} = \frac{0.7016}{3*9.8}(9.8*270) = 63.15\ s$

Solving for ΔV_G:

Making the substitution $z = \frac{t}{t_{b1}}$

$\Delta V_G = \int_0^{t_{b1}} g\sin\gamma\, dt = g t_{b1}\int_0^1 (1-z)dz = \frac{1}{2}g t_{b1}$ (7)

As a first approximation:

$\Delta V_G = \frac{1}{2} \times 9.8 \times 63.15$

$\Delta V_G = 309\frac{m}{s}$

We can now calculate a better ΔV_1:

$\Delta V_1 = \frac{1}{2}(6145 - 435 + 309 + 150)$

$\Delta V_1 = 3085 \frac{m}{s}$

Refine other quantities:

$\frac{\mu_{p1}}{\mu_{01}} = 1 - e^{-\frac{3085}{2646}} = 0.6884$

$t_{b1} = \frac{0.6884}{3} 270 = 61.95\ s$

$\Delta V_G = \frac{1}{2} \times 9.8 \times 61.95 = 304 \frac{m}{s}$

$\Delta V_1 = 3082 \frac{m}{s}$

$\Delta V_2 = \Delta V_1 = 3082 \frac{m}{s}$

These values are close enough to the first approximation, and we accept them as converged.

c) Calculation of Stage Masses:

For each stage:

$\frac{\mu_{pay,i}}{\mu_{0,i}} = e^{-\frac{\Delta V_i}{c}} - \varepsilon$ (8)

We apply this first to the 3rd stage, for which $m_{pay,3} = m_{pay} = 3\ kg$.

$M_{03} = \frac{3}{e^{-\frac{2257}{2646}} - 0.1} = \frac{3}{0.3361} = 9.20\ kg$

The structural mass of the third stage is then:

$M_{s3} = 0.1 M_{03} = 0.92\ kg$

The propellant mass is:

$M_{p3} = 9.20\left(1 - e^{-\frac{2257}{2646}}\right) = 5.28\ kg$

As a check: $M_{s3} + M_{p3} + M_{pay3} = 0.92 + 5.28 + 3 = 9.20\ kg = M_{03}$ (as it should)

For the second stage:

$M_{pay2} = M_{03} = 9.20\ kg$

$M_{02} = \frac{9.20}{e^{-\frac{3082}{2646}} - 0.1} = \frac{9.20}{0.2120} = 43.39\ kg$

$M_{s2} = 0.1 \times 43.39 = 4.34\ kg$

$M_{p2} = 43.39\left(1 - e^{-\frac{3082}{2646}}\right) = 29.85\ kg$

Again, we check that: $M_{s2} + M_{p2} + M_{pay2} = 4.34 + 29.85 + 9.20 = 43.39\ kg = M_{02}$

For the first stage:

$M_{pay4} = M_{02} = 43.49\ kg$

$M_{01} = \frac{43.39}{e^{-\frac{3082}{2646}} - 0.1} = \frac{43.39}{0.2120} = 204.67\ kg$

$M_{s1} = 0.1 \times 204.67 = 20.47\ kg$

$M_{p1} = 204.67\left(1 - e^{-\frac{3082}{2646}}\right) = 140.82\ kg$

We verify that $M_{s1} + M_{p1} + M_{pay1} = 20.47 + 140.82 + 43.39 = 204.68\ kg$

d) Thrusts and Firing Times:

$F_1 = M_{01} \times 3g = 204.67 \times 3 \times 9.8 = 6{,}017\ N$

$F_2 = M_{02} \times 3g = 43.39 \times 3 \times 9.8 = 1{,}276\ N$

$F_3 = M_{03} \times 3g = 9.20 \times 3 \times 9.8 = 270\ N$

Flow rates are then:

$\dot{m}_1 = \frac{6017}{2646} = 2.274\frac{kg}{s} \times\times$

$\dot{m}_2 = \frac{1276}{2646} = 0.4822\frac{kg}{s}$

$\dot{m}_1 = \frac{270}{2646} = 0.1020\frac{kg}{s}$

Firing times are given by $\boldsymbol{t_{bi} = \frac{m_{pi}}{\dot{m}_i}}$

$t_{b1} = \frac{140.82}{2.274} = 61.93\ s$

$t_{b1} = \frac{29.85}{0.4822} = 61.91\ s$

$t_{b1} = \frac{5.28}{0.1020} = 51.76\ s$

16.50 Introduction to Propulsion Systems
Spring 2012