16.410-13 Recitation 7 Problems

Problem 1: Propositional Logic

Consider the following English statement.

> *Julia says that Sarah and Fred say the truth. Sarah says that Peter is lying. Peter says that Fred says the truth. Fred says that Peter lies or Julia lies. Dick says, he lies.*

- *Write this sentence in propositional logic after defining appropriate atomic propositions.*

Solution: Let us define the atomic propositions, Julia, Sarah, and Fred to indicate that Julie is telling the truth, Sarah is telling the truth, and Fred is telling the truth, respectively.

First notice that if A lies then the opposite of A's statement is true, i.e., if A claims that B and C tell the truth but A lies, then we have "(not A) implies (not(B and C))", which is equivalent to "(not A) implies ((not B) or (not C))" by de Morgan's law. Now we can write the following several statements, each of which is true for the given English sentence.

A1. Julia implies (Sarah and Fred)

A2. (not Julie) implies ((not Sarah) or (not Fred))

A3. Sarah implies (not Peter)

A4. (not Sarah) implies Peter

A5. Peter implies Fred

A6. (not Peter) implies (not Fred)

A7. Fred implies ((not Peter) or (not Julia))

A8. (not Fred) implies (Peter and Julia)

A9. Dick implies (not Dick)

A10. (not Dick) implies Dick

The sentence is basically a conjunction of the all the statements above, i.e.,

> (Julia implies (Sarah and Fred)) and ((not Julie) implies ((not Sarah) or (not Fred))) and (Sarah implies (not Peter)) and ((not Sarah) implies Peter) and (Peter implies Fred) and ((not Peter) implies (not Fred)) and (Fred implies ((not Peter) or (not Julia))) and ((not Fred) implies (Peter and Julia)) (Dick implies (not Dick)) and ((not Dick) implies Dick)

- *Determine those people who are lying and those who are telling the truth.*

Solution: Let's pick some person and assume that she/he is lying. Let this person be Peter. Then we can derive the following:

B1. (not Peter) *Our hypothesis.*

B2. (not Fred) *(A6) and (B1).*

B3. Peter and Julia *(A8) and (H2).*

B4. Peter *(B3), i.e., (Peter and Julia) implies Peter.*

Notice that (B1) and (B4) contradict each other. Thus, Peter must be saying the truth. Now we can conclude the following:

C1. Peter *from the derivation B. above*

C2. Fred *(A5) and (C1)*

C3. (not Peter) or (not Julia) *(A7) and (C2)*

C4. Peter implies (not Julia) *implication form of (C3)*

C5. not Julia *(C1) and (C4)*

C6. (not Sarah) or (not Fred) *(A2) and (A15)*

C7. Fred implies (not Sarah) *implication form of (C6)*

C8. (not Sarah) *(C2) and (C7).*

Hence, we conclude that both Julia and Sarah are lying, and that Peter and Fred tell the truth.

- *What can you tell about Dick?*

Solution: Dick is an interesting situation. In English, if he is telling the truth, then he claims that he is lying. But, on the other hand, if he is lying, then he claims that he is telling the truth. Hence, Dick does not have a truth value in propositional logic. In other words, what he is saying is contradictory.

Problem 2: Conjunctive Normal Form

Put the following statements into conjunctive normal form:

- *(A iff B) and (B iff A)*

Solution:

(A iff B) and (B iff A)
(A implies B) and (B implies A)
((not A) or B) and ((not B) or A).

Hence, we have the following conjuncts:
C_1: ((not A) or B)
C_2: ((not B) or A)

- *(A implies (not (B or C))) and (A iff B)*

Solution:

(A implies (not (B or C))) and (A iff B)
((not A) or (not (B or C))) and ((A implies B) and (B implies A))
((not A) or (not (B or C))) and (((not A) or B) and ((not B) or A))
((not A) or ((not B) and (not C))) and (((not A) or B) and ((not B) or A))
(((not A) or (not B)) and ((not A) or (not C))) and (((not A) or B) and ((not B) or A)).

Hence, we have the following conjuncts:
C_1: (not A) or (not B)
C_2: (not A) or (not C)
C_3: (not A) or B
C_4: (not B) or A.

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