16.346 Astrodynamics
Fall 2008

Exercises 10

1. Problem 6–28 The Lagrange parameters α_m and β_m for the minimum-energy orbit are

$$\alpha_m = \pi \qquad \cos\beta_m = \frac{3c - r_1 - r_2}{r_1 + r_2 + c}$$

and the transfer time is

$$\sqrt{\frac{\mu}{a_m^3}}(t_2 - t_1) = \pi - (\beta_m - \sin\beta_m)$$

2. Problem 6–30 The transfer time for an elliptic arc with vacant focus F^* connecting points P_1 and P_2, after it has been transformed to a rectilinear ellipse, can be calculated from the one-dimensional form of the vis-viva integral

$$v^2 = \left(\frac{dr}{dt}\right)^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

Derive the equation

$$\sqrt{\mu}(t_2 - t_1) = \int_{s-c}^{s} \frac{r\,dr}{\sqrt{2r - r^2/a}}$$

and carry out the integration using the following change of variable

$$r = a(1 - \cos x)$$

to obtain Lagrange's equation

$$\sqrt{\mu}(t_2 - t_1) = a^{\frac{3}{2}}[(\alpha - \sin\alpha) - (\beta - \sin\beta)]$$

where α and β are angles in the upper half-plane.

In a two-body orbital boundary-value problem, the initial and terminal position vectors are $\mathbf{r}_1 = 4\,\mathbf{i}_x$ and $\mathbf{r}_2 = 4\,\mathbf{i}_x + 3\,\mathbf{i}_y$.

3. Calculate

- **a.** The minimum possible eccentricity for an elliptic orbit.
- **b.** The semimajor axis of the minimum energy orbit.
- **c.** The eccentricity vector for the minimum energy orbit and its eccentricity.
- **d.** The eccentricity vector the minimum energy orbit.
- **e.** The parameter of the minimum energy orbit.

4. The velocity vector at the initial point is $\mathbf{v}_1 = 4\,\mathbf{i}_x + 3\,\mathbf{i}_y$. Calculate

- **a.** The velocity vector $\mathbf{v}_2$ at the terminal point.
- **b.** The gravitational constant μ.

5. For the minimum energy orbit, calculate

- **a.** The time of flight from $\mathbf{r}_1$ to $\mathbf{r}_2$.
- **b.** The velocity vector at the initial point.