16.346 Astrodynamics
Fall 2008

Exercises 05

An extraordinary alignment of the planets occurred on June 17, 1991.

Try the Solar System Simulator on the internet at (space.jpl.nasa.gov)

1. Plot the positions of the four planets: Venus, Earth, Mars and Jupiter for this date corresponding to Julian day number 2,448,424. For convenience, assume the planets are coplanar. Use the tabulated data for the epoch 1960 January 1.5 ephemeris time corresponding to Julian day number 2,436,935:

Planet	Ω deg	ϖ deg	ϵ deg	e	a a.u.	i deg
Mercury	47.85714	76.83309	222.62165	0.205627	0.387099	7.00399
Venus	76.31972	131.00831	174.29431	0.006793	0.723322	3.39423
Earth		102.25253	100.15815	0.016726	1.000000	0.0
Mars	49.24903	335.32269	258.76729	0.093368	1.523691	1.84991
Jupiter	100.04444	13.67823	259.83112	0.048435	5.202803	1.30536
Saturn	113.30747	92.26447	280.67135	0.055682	9.538843	2.48991
Uranus	73.79630	170.01083	141.30496	0.047209	19.181951	0.77306
Neptune	131.33980	44.27395	216.94090	0.008575	30.057779	1.77375

See also (http://ssd.jpl.nasa.gov/) for the Planetary Orbital Elements for which the epoch is J2000 or 2000 January 1.5

2. Show that the equations for the position and velocity vectors in reference coordinates for zero inclination angle are

$$\mathbf{r} = r\cos L\,\mathbf{i}_x + r\sin L\,\mathbf{i}_y$$
$$\mathbf{v} = -\frac{\mu}{h}(\sin L + e\sin\varpi)\,\mathbf{i}_x + \frac{\mu}{h}(\cos L + e\cos\varpi)\,\mathbf{i}_y$$

where $L = \varpi + f$ is the true longitude and $\varpi = \Omega + \omega$ is the longitude of pericenter.

3. Verify the mean anomalies, expressed in radians, of the planets for the given date are:

$$M_V = 1.573746963 \quad M_E = 2.81988346 \quad M_M = 3.212448431 \quad M_J = 2.108102726$$

Use the following tabulated values for the mean motion n:

$$n_V = 1.602131\,^\circ/\text{day} \quad n_E = 0.985609\,^\circ/\text{day} \quad n_M = 0.524033\,^\circ/\text{day} \quad n_J = 0.083091\,^\circ/\text{day}$$

4. The position and velocity vectors of a spacecraft are

$$\mathbf{r} = -\frac{1}{\sqrt{2}}(35\,\mathbf{i}_x + 4\,\mathbf{i}_y + 3\mathbf{i}_z)$$
$$\mathbf{v} = -\frac{1}{25\sqrt{2}}(5\,\mathbf{i}_x + 4\,\mathbf{i}_y + 3\mathbf{i}_z)$$

with units so chosen that the gravitational constant μ is unity. Determine the parameter, the eccentricity, the longitude of the ascending node, the argument of pericenter, the inclination and the eccentric anomaly of the orbit.

5. The position vector of a spacecraft is

$$\mathbf{r} = 3\,\mathbf{i}_x + 4\,\mathbf{i}_y$$

If the eccentricity vector is $\mathbf{e} = \frac{1}{2}\,\mathbf{i}_z$, find the parameter of the orbit and the eccentric anomaly of the spacecraft.

6. A transit of Venus across the sun took place on June 8, 2004. The last time it happened was 121 years ago. Calculate the positions of the Earth and Venus on the day of the most recent transit.

Here again use the Solar System Simulator on the internet at (space.jpl.nasa.gov).

Euler is buried in St. Petersburg (Petrograd), Russia in a cemetery with all the famous Russians. He is there because he was a protege of the Empress Catherine the Great.