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16.323 Principles of Optimal Control  
Spring 2008

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## 16.323 Homework Assignment #1

Any code for this homework should be submitted online using the class web page - do not forget, as it will be graded as part of your solution.

1. For the following function,

$$F(x_1, x_2, x_3) = 2x_1^4 + 3x_2^2 + 6x_3^2 - 3x_1x_2 - 6x_2x_3$$

- (a) Find the minimum(s).
  - (b) Are there any other stationary points? If so, what are they?
2. Give the criteria used to determine if a symmetric real matrix is negative definite or positive semidefinite, in terms of 1) eigenvalues and 2) determinants of respective submatrices.

3. This problem explores the steepest descent algorithm. For the following function,

$$F(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3 - 2x_1 - 8x_3$$

- (a) Give an expression for the search direction  $\mathbf{p}_k$  for the steepest descent method.
  - (b) Using  $\mathbf{x}_0 = [1 \ 1 \ 1]^T$ , write out the first iteration of the steepest descent algorithm. What is  $\mathbf{x}_1$  in terms of  $\alpha_0$ ? What is the optimal value for  $\alpha_0$  that minimizes  $F(\mathbf{x}_0 + \alpha_0\mathbf{p}_0)$ ?
  - (c) Write a MATLAB program to solve this problem using a steepest descent algorithm and an initial value of  $\mathbf{x}_0 = [1 \ 1 \ 1]^T$ . Using a tolerance of  $10^{-6}$ , how many iterations does it take to converge with  $\alpha = 0.1$ ,  $\alpha = 0.2$ ,  $\alpha = 0.5$ ,  $\alpha = 1$ ? Explain your results.
  - (d) Use MATLAB's `fminunc` function to solve for the minimum and compare its performance with your algorithm.
4. For the following cost function,  $F = x^2 + y^2 - 6xy - 4x - 5y$

- (a) Show analytically how to minimize the cost subject to the constraints,

$$f_1 : -2x + y + 1 \geq 0$$

$$f_2 : x + y - 4 \leq 0$$

$$f_3 : x \geq -1$$

(b) How is the optimal cost affected if the constraint  $f_1$  is changed to,

$$f'_1 = -2x + y + 1.1 \geq 0$$

Estimate this difference and explain your answer.

(c) Write a Matlab script to confirm your results in parts (a) and (b)

5.

Content from Bryson, page 12. Removed due to copyright restrictions.

6. Read the article on the web by Bryson on the history of optimal control and write a short summary (approximately 1/2 page). In particular, identify some of the key players and the main algorithmic/technological steps made in those 35 years.