Handout 4: Root-Locus Review

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Summary of Guidelines for plotting a root-locus

1. Mark Poles **X** and Zeros **O**.
2. Draw the locus on the real axis to the left of an odd number of real poles plus zeros.
3. Draw $n - m$ asymptotes (n is the number of poles, m the number of zeros). The asymptotes are centered at α and leave at angles Φ_l, where

$$\alpha = \frac{\sum p_i - \sum z_i}{n-m} = \frac{-a_1 + b_1}{n-m},$$
$$\phi_l = \frac{180^o + l360^o}{n-m}, \quad l = 0, 1, 2, \ldots n - m - 1.$$

4. Compute the loci departure angles from the poles and arrival angles at the zeros.
5. Assume $s_0 = j\omega_0$ and compute the point(s) where the locus crosses the imaginary axis for positive K.
6. The equation has multiple roots at points on the locus where

$$b\frac{da}{ds} - a\frac{db}{ds} = 0.$$

   If s_0 is on the real axis, these points are points of breakaway or break-in. Compute the angles of arrival and the angles of departure for any points of multiple roots.
7. Complete the locus, using the previous steps *and your experience*.

$$G(s) = \frac{s+1}{s^2(s+4)}$$

$$G(s) = \frac{s+1}{s^2(s+12)}$$

$$G(s) = \frac{(s+0.1)^2 + 16}{s((s+0.1)^2 + 25)}$$