Lecture 32 Dec 1, 03.

3D Boundary Layers.

9-1) Introduction
New Effects: Cross flow
Lateral Dilation

<u>Reading</u>

A) Introduction: 3D TSL steady, compressible or incompressible, viscous.

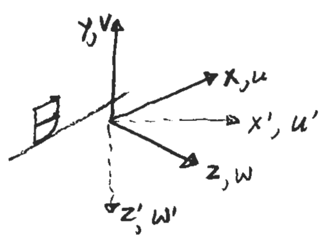
Mass: $$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X - Mom: $$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}$$

Z - Mom: $$\rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{zy}}{\partial y}$$ ← New eqn.

Y - Mom: $$0 \approx \frac{\partial p}{\partial y}$$

Note that above equations are rotationally invariant in y.



$$\bar{\bar{\tau}} = \begin{bmatrix} 0 & \tau_{xy} & 0 \\ \tau_{xy} & 0 & \tau_{yz} \\ 0 & \tau_{zy} & 0 \end{bmatrix} \rightarrow \bar{\tau}' = \begin{bmatrix} 0 & \cdot & \\ & 0 & \cdot \\ \cdot & \cdot & 0 \end{bmatrix}$$

From Bernoulli's eqn we have in the free stream

$$-\frac{\partial p_e}{\partial x} = \rho_e U_e \frac{\partial U_e}{d x} + \rho_e W_e \frac{\partial U_e}{\partial z}$$

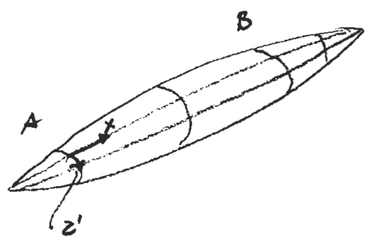
$$-\frac{\partial p_e}{\partial z} = \rho_e U_e \frac{\partial W_e}{\partial x} + \rho_e W_e \frac{\partial W_e}{\partial z}.$$

The surface stream line is at an angle β w.r.t the main stream direction

$$\tan \beta = \frac{\tau_{zy}}{\tau_{xy}} = \lim_{y \to 0} \frac{W}{U}$$

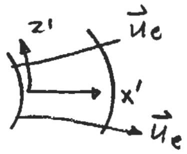
B) New Effects

① Lateral dilation : ex. on a body of revolution



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial z} \rightarrow \text{like a source term}$$

At A

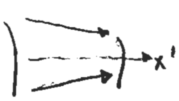


streamlines moving apart $\frac{\partial w}{\partial z} > 0$

mass disappearing in crossflow plane.

main BL grows more slowly.

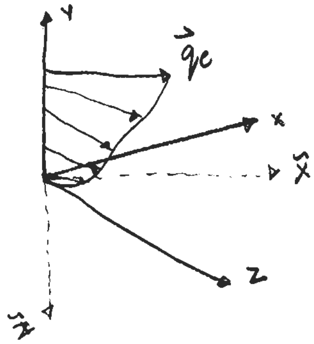
B



streamlines converging ($\frac{\partial w}{\partial z} < 0$, mass appearing)

main BL grows more rapidly

(2) Cross-flow. appearance of

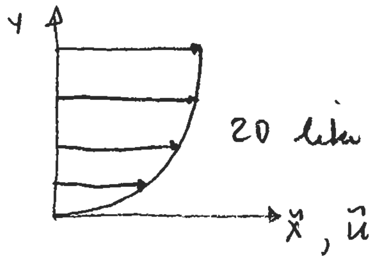


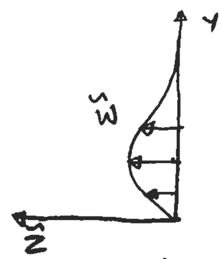
$\hat{U}_e$ is locally aligned with free stream direction

$\hat{x} \parallel \vec{q}_e$, $\hat{z} \perp \vec{q}_e$

$$\hat{U}_e = \|\vec{q}_e\|$$

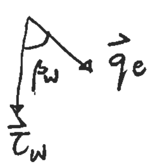
$$\hat{W}_e = 0$$





Wall shear stress angle β_w

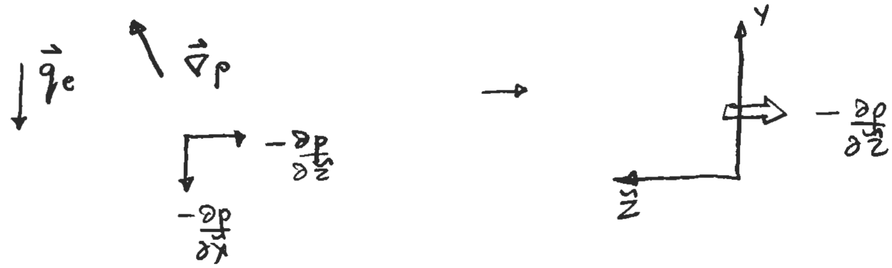
$$\tan\beta_w = \lim_{y \to 0} \frac{w}{u} = \lim_{y \to 0} \tau_{zy}/\tau_{xy}$$



Reasons for cross flow: Two ways to look at it

① Kinematic explanation : vortex tilting (not strict ∵ vorticity not conserved)

② Dynamic " : transverse pressure gradient.



Outside BL

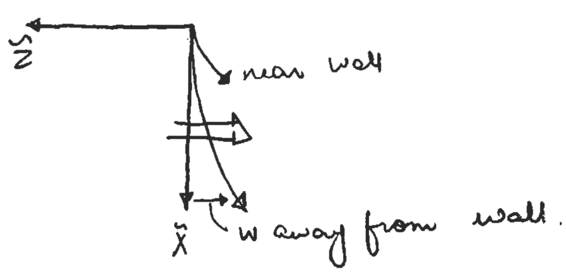
$$\frac{\partial p}{\partial \tilde{z}} = -\frac{\rho \tilde{u}_e^2}{R} = -\frac{\rho q_e^2}{R_e} \quad \text{(normal acceleration)}$$

Inside BL

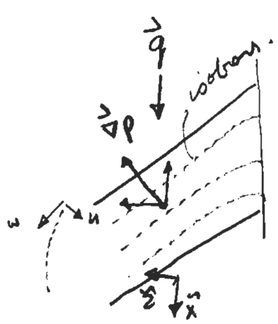
$$\frac{\partial p}{\partial \tilde{z}} = \not{c} \ \text{in } y \approx -\frac{\rho q^2}{R'} \approx -\rho \tilde{u} \frac{\partial \tilde{w}}{\partial \tilde{x}}$$

where $q^2 < q_e^2 \Rightarrow R' < R_e$

more curvature in BL



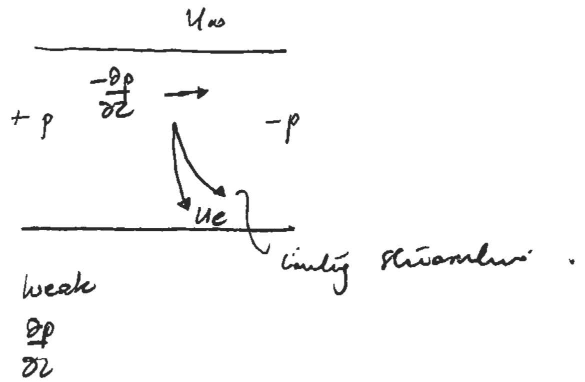
Example: swept wing

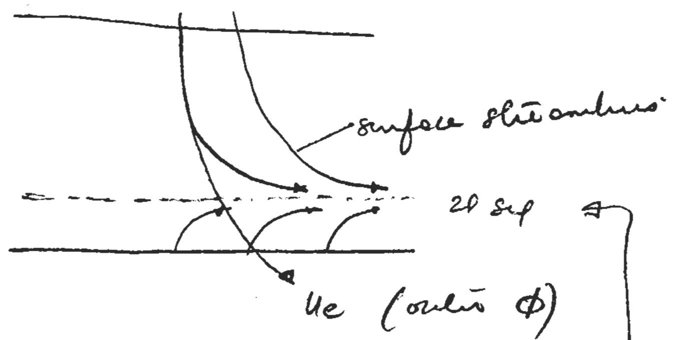


3D Separation:

General 3D separation summarized in White. We examine special case of separation on infinite yawed wing/cylinder.

Important Effect: low speed fluid near wall responds strongly to cross-pressure gradients





surface streamlines approach asymptotically to separation line

2D slice

Boundary sheet separates and becomes vortex sheet.