Turbulent Shear layers.

7.1) B) Prandtl's Analogy

7.2) A) Turbulent BL Structure

B) Effect of Roughness

Reading: Sch 495 - 552
l&B 160 - 210
Wh 394 - 449.

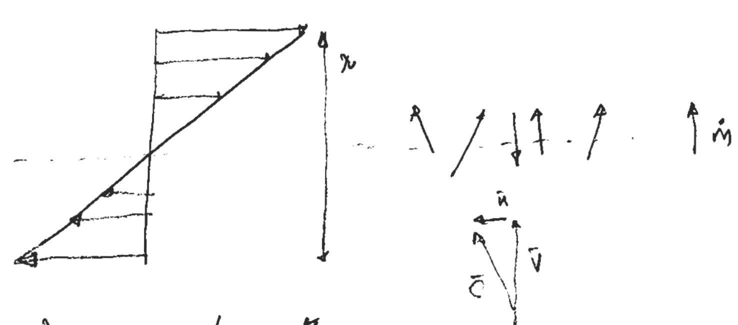
7.1 B) Prandtl's Analogy (with kinetic theory)

from last class.

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho}\frac{\partial}{\partial x}\big(\bar{p} + \underbrace{\rho\overline{u'^2}}_{\sigma_x'}\big) + \frac{\partial}{\partial y}\underbrace{\left(\mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'}\right)}_{\tau} \qquad \text{– 2D X-mom}$$

TSL

Molecular Level



λ = mean free path

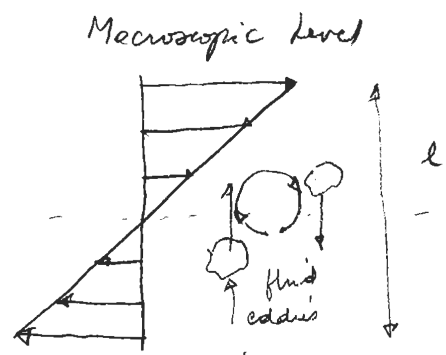
c̄ = molecular speed

X-mom flux: $\dot{m}\bar{u} = \rho\bar{v}\bar{u} = \rho\bar{v}\frac{\lambda}{2}\frac{d\bar{u}}{dy}$

$$\approx \underbrace{\rho\bar{c}\frac{\lambda}{2}}_{\mu}\frac{d\bar{u}}{dy}$$

Y-mom flux: $p \approx \rho\bar{c}^2$

Macroscopic Level

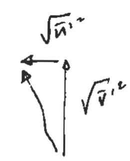


l = mixing length

$\tau_{turb} = -\rho\overline{u'v'}$

$p_{turb} \approx \rho\overline{u'^2}$

$\sqrt{\overline{u'^2}} \approx \sqrt{\overline{v'^2}}$



Similar to laminar (molecular level)
Analogous.

$$|\overline{u'}| = l \left| \frac{d\bar{u}}{dy} \right|$$

Since $v' \sim u'$ magnitude

$$|\overline{v'}| = \text{const.} \, |\overline{u'}| = \text{const.} \, l \left| \frac{d\bar{u}}{dy} \right|$$

Approximate

$$\overline{u'v'} = -c \, |\overline{u'}| \, |\overline{v'}| \qquad 0 < c < 1 \quad (\text{constant})$$

$$\therefore \quad \overline{u'v'} = -\text{const} \, l^2 \left(\frac{d\bar{u}}{dy} \right)^2$$

$$\therefore \quad \tau_t = -\rho \overline{u'v'} = \rho l^2 \left(\frac{d\bar{u}}{dy} \right)^2$$

Where l is an unknown mixing length (absorb constants)

A more correct way is to write

$$\tau_t = \rho l^2 \left| \frac{d\bar{u}}{dy} \right| \left(\frac{\partial \bar{u}}{dy} \right)$$

since τ_t can change sign with $\frac{d\bar{u}}{dy}$.

Comparing with Boussinesq hypothesis

$$\tau_t = \mu_t \frac{d\bar{u}}{dy}$$

we get

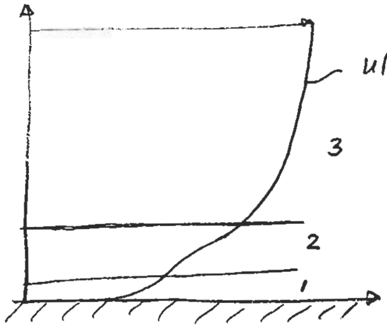
$$\mu_t = \rho l^2 \left| \frac{d\bar{u}}{dy} \right|$$

or $\nu_t = l^2 \left| \frac{d\bar{u}}{dy} \right|$ — Mixing length model

Complete if we can relate l to the flow.

7.2) A) Turbulent BL Structure

Turbulent wall BL has 3 distinct layers (emp. fact)

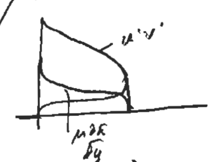


wake / outer layer } outer

← log layer
← laminar sublayer } wall / inner layer

unlike laminar BL
(
· scales

* Inner layer – viscous shear (μ molecular) dominates } overall – transition both imp ⇒ different. struc
* Outer layer – turbulent shear dominates



Inner layer retards outer wake layer

In the inner layer

$$u = f(\underbrace{\tau_w, \rho, \mu}_{\text{physical params.}}, y, k)$$ — functional depend

↑ roughness height

Velocity scale : $u_\tau = \sqrt{\frac{\tau_w}{\rho}} = u^*$ (shear velocity) (no u_e, δ)

Length scale : $l_\tau = \frac{\mu}{\sqrt{\rho \tau_w}} = \frac{\nu}{u^*} = l^*$

Non-dimension dep : $\frac{u}{u^*} = f(y/l^*, k/l^*)$

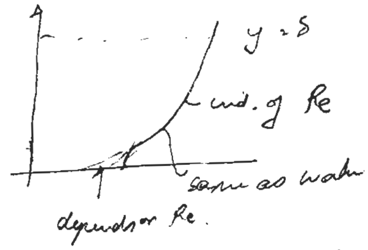
↓

$\underline{\underline{y/l^* = y^+}}$, $\underline{\underline{u/u^* = u^+}}$ (inner variables)

In outer layer : velocity scale $u_e - u$

defect law → $U_e - \bar{u} = f(y, \rho, \tau_w, \overset{\sim \delta}{\delta^*}, \frac{dp}{dx})$

Length scale : δ or δ^*



Non-dimension :

$$\frac{u_e - \bar{u}}{u^*} = g\left(y/\delta^*, \frac{\delta^*}{\tau_w}\frac{dp}{dx} \right)$$

→ later

↓ β non-dimensional pressure gradient.

velocity defect is dependant variable.

We can develop velocity profile

① Sublayer

$$u^+ = u/u^* \qquad y^+ = y/L^* \qquad \therefore \mu \frac{\partial \bar{u}}{\partial y} = \tau_w \Rightarrow \frac{\partial u^+}{\partial y^+} = 1$$

$$\therefore \boxed{u^+ = y^+} \text{ in viscous sublayer}$$

② Over lap / log layer

$$u^+ = f(y^+) = f\left(\left(\frac{\delta v^*}{\nu}\right) \cdot (y/\delta)\right) = \frac{U_e}{U^*} - g(y/\delta)$$

f contains multiplicative const , g an additive const.

$\Rightarrow$ f and g are logarithmic

$$u^+ = \frac{1}{\kappa} \ln y^+ + B$$

$$\frac{U_e - u}{u^*} = -\frac{1}{\kappa} \ln (y/\delta) + A$$

↑ depends on $\frac{dp}{dx}$

Other approach : $l = \kappa y$ (mixing length)

$$\tau = \rho l^2 \left(\frac{\partial \bar{u}}{\partial y}\right)^2 = \rho \kappa^2 L^2 \left(\frac{\partial \bar{u}}{\partial y}\right)^2$$

Near the wall : $\tau \approx \tau_w \rightarrow \tau_w/\rho = u^{*2} = \kappa^2 y^2 \left(\frac{\partial \bar{u}}{\partial y}\right)^2$.

Integrating

$$\bar{u} = \frac{u^*}{k} \ln \underset{(y^+)}{y} + C$$

At $y = \delta$ $\bar{u} = u_e$

$$\therefore \quad \frac{\bar{u} - u_e}{u^*} = -\frac{1}{k} \ln (y/\delta) \qquad \text{(0 pressure gradient)}$$