Turbulent Shear Layers

1.1) A) Reynolds Averaging

B) Prandtl's Ananalogy

Reading: Sch 496 – 538 ~~555 – 571~~

Wh 394 – 463

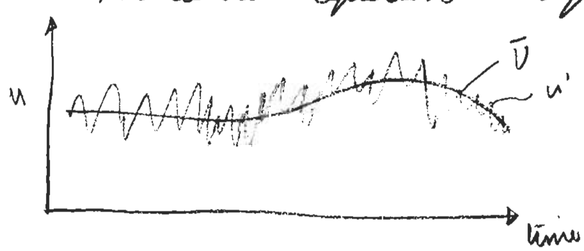
C&B

A) Turbulent Closure

Turbulent flow characterized by random fluctuations.
Recall we substituted perturbation flowfield into the incompressible mass and momentum equations

	$\bar{u}, \bar{v}$	u', v'	u'^2, v'^2, etc
Linear Stability Th.	known (laminar)	unknown (exponential)	drop. $u' \ll \bar{u}$, etc
Turbulent Flow	unknown (not laminar)	eliminate via Rey avg	unknown require addl info Ex – turb model

Additional equations required to solved for turbulent flow in addition to mass, momentum & energy.



B) Reynolds Avg.

Separate flowfield (velocity and pressure) into mean and fluctuating components. Let introduces averaging procedures.

Time Averaging $\quad \overline{(\ )} \equiv \frac{1}{T}\int_{t_0}^{t_0+T} (\ )\,dt \quad$ (fixed point in space) when T is sufficiently large so that $\bar{(\ )}$ is indep. of time

Ensemble Avg $\quad \langle (\ ) \rangle \equiv \lim\limits_{N \to \infty} \frac{1}{N} \sum\limits_{i=1}^{N} (\ )_i$

· can be unsteady.

In practice, $\bar{u} = \langle u \rangle$, $\bar{v} = \langle v \rangle$, etc. if u, v, etc are <u>ergodic</u>, i.e. statistically constant in time

Note a few rules of operating on time averages.

$$f = \bar{f} + f' \qquad g = \bar{g} + g'$$

then

$$\overline{f'} = \overline{g'} = 0 \qquad \overline{f'\bar{g}} = 0$$

$$f^2 = (\bar{f} + f')^2 = \bar{f}^2 + 2\bar{f}f' + f'^2$$

$$\overline{f \cdot g} = \bar{f}\bar{g} + \overline{f'g'}$$

Applying to the flowfield

$$u = \bar{u} + u'$$
$$v = \bar{v} + v'$$
$$w = \bar{w} + w'$$

$$p = \bar{p} + p'$$

Mass conservation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial(\bar{u}+u')}{\partial x} + \frac{\partial}{\partial y}(\bar{v}+v') + \frac{\partial}{\partial z}(\bar{w}+w') = 0$$

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} + \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial z} = 0$$

Taking the time average we get

$$\frac{\partial \overset{=\bar{\bar{u}}}{\bar{u}}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \quad \text{since } (\bar{f'} = 0)$$

which implies fluctuations satisfy continuity.

$u \cdot$ mass + x - mom

$$u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y} + u\frac{\partial w}{\partial z} + \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\Rightarrow \quad \frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

Substitute velocity decomp

$$\frac{\partial \bar{u}+u'}{\partial t} + \frac{\partial}{\partial x}(\bar{u}+u')^2 + \frac{\partial}{\partial y}(\bar{u}+u')(\bar{v}+v') + \frac{\partial}{\partial z}(\bar{u}+u')(\bar{w}+w')$$

$$= -\frac{1}{\rho}\frac{\partial}{\partial x}(\bar{p}+p') + \nu\nabla^2(\bar{u}+u')$$

$$\frac{\partial \bar{u}}{\partial t} + \frac{\partial u'}{\partial t} + \frac{\partial}{\partial x}\bar{u}^2 + 2\frac{\partial}{\partial x}(\bar{u}u') + \frac{\partial}{\partial x}(u')^2 + \frac{\partial}{\partial y}(\bar{u}\bar{v}) + \frac{\partial}{\partial y}(\bar{u}v') + \frac{\partial}{\partial y}(u'\bar{v})$$

$$+ \frac{\partial}{\partial y}(u'v') + \frac{\partial}{\partial z}(\quad) \cdots\cdots + \frac{\partial}{\partial z}(u'w')$$

$$= -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} - \frac{1}{\rho}\frac{\partial p'}{\partial x} + \nu\nabla^2\bar{u} + \nu\nabla^2 u'$$

Take time average

$$\rightarrow \underbrace{\frac{\partial \bar{u}}{\partial t} + \frac{\partial}{\partial x}(\bar{u}^2) + \frac{\partial}{\partial y}(\bar{u}\bar{v}) + \frac{\partial}{\partial z}(\bar{u}\bar{w}) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \nu \nabla^2 \bar{u}}_{\text{Laminar flow}}$$

$$- \underbrace{\left[\frac{\partial}{\partial x}(\overline{u'^2}) + \frac{\partial}{\partial y}(\overline{u'v'}) + \frac{\partial}{\partial z}(\overline{u'w'}) \right]}_{\text{new unknowns}}$$

→ Reynolds or apparent stresses.

Rewrite in traditional form

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \cdots \bar{w}\frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial x}\left(\nu \frac{\partial \bar{u}}{\partial x} - \overline{u'^2}\right)$$

$$+ \frac{\partial}{\partial y}\left(\nu \frac{\partial \bar{u}}{\partial y} - \overline{u'v'}\right)$$

$$+ \frac{\partial}{\partial z}\left(\nu \frac{\partial \bar{u}}{\partial z} - \overline{u'w'}\right)$$

$$* \quad (\quad) = \cdots$$

We can write

dominant term — turbulent shear

$$\frac{\partial}{\partial y}\left(\nu \frac{\partial \bar{u}}{\partial y} - \overline{u'v'}\right) = \frac{\partial}{\partial y}\left((\nu + \nu_t)\frac{\partial \bar{u}}{\partial y}\right)$$

where $\nu_t = -\dfrac{\overline{u'v'}}{\frac{\partial \bar{u}}{\partial y}}$ ← Eddy Viscosity (100 μ)

which is a property of the flow field (not fluid)

$$\begin{bmatrix} \sigma_x' & \tau_{xy}' & \tau_{xz}' \\ \tau_{xy}' & \sigma_y' & \tau_{yz}' \\ \tau_{xz}' & \tau_{yz}' & \sigma_z' \end{bmatrix} = - \begin{bmatrix} \rho\overline{u'^2} & \rho\overline{u'v'} & \rho\overline{u'w'} \\ \rho\overline{u'v'} & \rho\overline{v'^2} & \rho\overline{v'w'} \\ \rho\overline{u'w'} & \rho\overline{v'w'} & \rho\overline{w'^2} \end{bmatrix}$$

(*) due to macroscopic-level momentum transport (mixing)

2-D Turbulent, incompressible, flow – same approximations as TBL for laminar flow, $\frac{\partial}{\partial z} \to 0$, $\bar{w} = 0$, we get

$$\nabla \cdot \vec{\bar{u}} = 0$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = \underbrace{u_e \frac{du_e}{dx}}_{\text{Bernoulli}} + \underbrace{\frac{1}{\rho}\frac{\partial \bar{\tau}}{\partial y}}_{\text{different}}$$

$$\tau = \mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'}$$

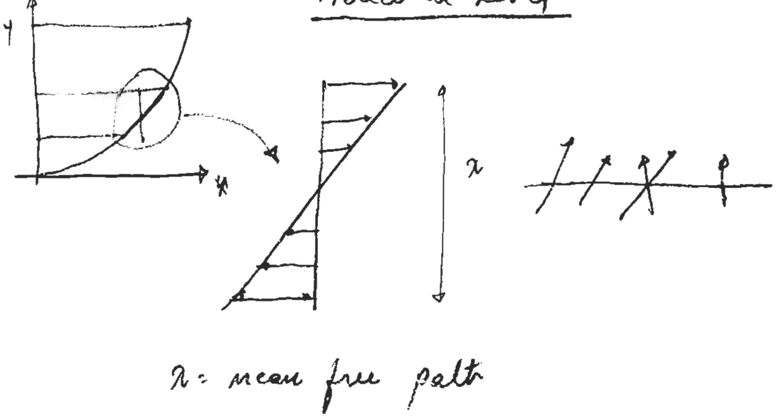
Y – mom

$$\frac{\partial p}{\partial y} \approx -\rho \frac{\partial}{\partial y}\overline{v'^2} \longrightarrow p = p_e(x) - \underbrace{\cancel{\rho \overline{v'^2}}}_{\text{small}}$$

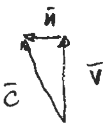
Same no slip condition holds at the wall and free stream matching at edge of BL $y = \delta$.

B) Prandtl's analogy.

Molecular Level



λ = mean free path

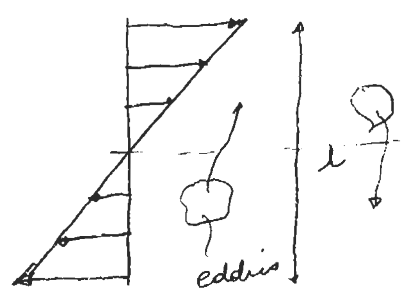


$\mu = \frac{1}{2}\rho \bar{c} \lambda$

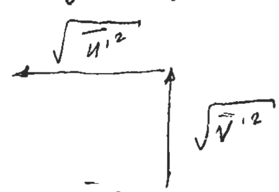
$p \sim \rho \bar{c}^2$

$$\tau = \mu \frac{\partial u}{\partial y} \sim \rho \underbrace{\bar{u}}_{\lambda \frac{\partial u}{\partial y}} \underbrace{\bar{v}}_{\sim \bar{v}} \sim \rho \bar{c} \lambda \frac{\partial u}{\partial y}$$

Macroscopic Level



l = mixing length



$P_{turb} = \rho \overline{u'^2}$ — x-dir

$\tau_{xy\,turb} = -\rho \overline{u'v'}$