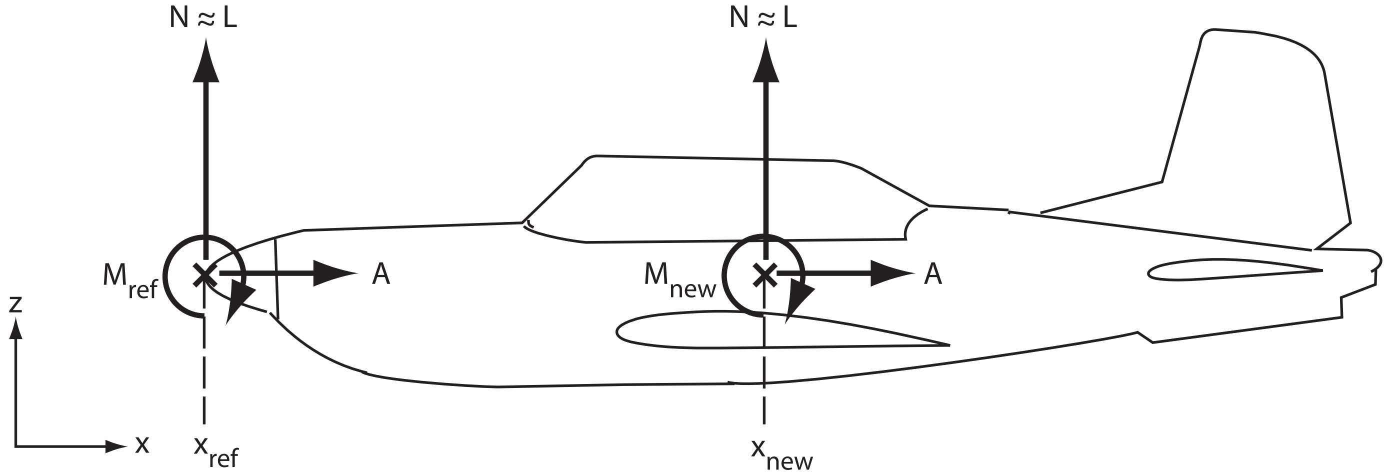
Aerodynamic Center[1]

Suppose we have the forces and moments specified about some reference location for the aircraft, and we want to restate them about some new origin.



$M_{ref} =$ Pitching moment about x_{ref}

$M_{new} =$ Pitching moment about x_{new}

$x_{ref} =$ Original reference location

$x_{new} =$ New origin

$N =$ Normal force $\approx L$ for small $\propto$

$A =$ Axial force $\approx D$ for small $\propto$

Assuming there is no change in the z location of the two points:

$$M_{ref} = -(x_{new} - x_{ref})L + M_{new}$$

Or, in coefficient form:

$$C_{m_{ref}} = -\left(\frac{x_{new} - x_{ref}}{\underbrace{\bar{c}}_{mean\ a.c.}} \right) C_L + C_{M_{new}}$$

The *Aerodynamic Center* is defined as that location x_{ac} about which the pitching moment doesn't change with angle of attack.

How do we find it?

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[1] Anderson 1.6 & 4.3

Let $x_{new} = x_{ac}$

Using above:

$$C_{M_{ref}} = -\frac{(x_{ac} - x_{ref})}{\bar{c}} C_L + C_{M_{ac}}$$

Differential with respect to α:

$$\frac{\partial C_{M_{ref}}}{\partial \alpha} = -\left(\frac{x_{ac} - x_{ref}}{\bar{c}}\right)\frac{\partial C_L}{\partial \alpha} + \left(\frac{\partial C_{M_{ac}}}{\partial \alpha}\right)$$

By definition:

$$\left(\frac{\partial C_{M_{ac}}}{\partial \alpha}\right) = 0$$

Solving for the above

$$\frac{x_{ac} - x_{ref}}{\bar{c}} = -\left(\frac{\partial C_{M_{ref}}}{\partial \alpha}\right)\Bigg/\left(\frac{\partial C_L}{\partial \alpha}\right),\ or$$

$$\boxed{\frac{x_{ac} - x_{ref}}{\bar{c}} = \frac{x_{ref}}{\bar{c}} - \left(\frac{\partial C_{M_{ref}}}{\partial C_L}\right)}$$

Example:

Consider our AVL calculations for the F-16C

- $x_{ref} = 0$ - Moment given about LE
- Compute $\frac{\partial C_{M_{ref}}}{\partial C_L}$ for small range of angle of attack by numerical differences. I picked $\alpha = -3^0 \text{ to } \alpha = 3^0$.
- This gave $\frac{x_{ac}}{\bar{c}} \approx 2.89$.
- Plotting $C_M\ vs\,\alpha$ about $\frac{x_{ac}}{\bar{c}}$ shows $\frac{\partial C_M}{\partial \alpha} \approx 0$.

Note that according to the AVL predictions, not only is $\frac{\partial C_M}{\partial \alpha} \approx 0\ @\ x_{ac} = 2.89$, but also that $C_M = 0$. The location about which $C_M = 0$ is called the *center of pressure*.

Center of pressure is that location where the resultant forces act and about which the aerodynamic moment is zero.

Changing “new” to “cp” and “ref” to “NOSE” to correspond to AVL:

$$C_{M_{NOSE}} = -\left(\frac{x_{cp} - x_{NOSE}}{\bar{c}}\right) C_L + C_{M_{cp}}$$

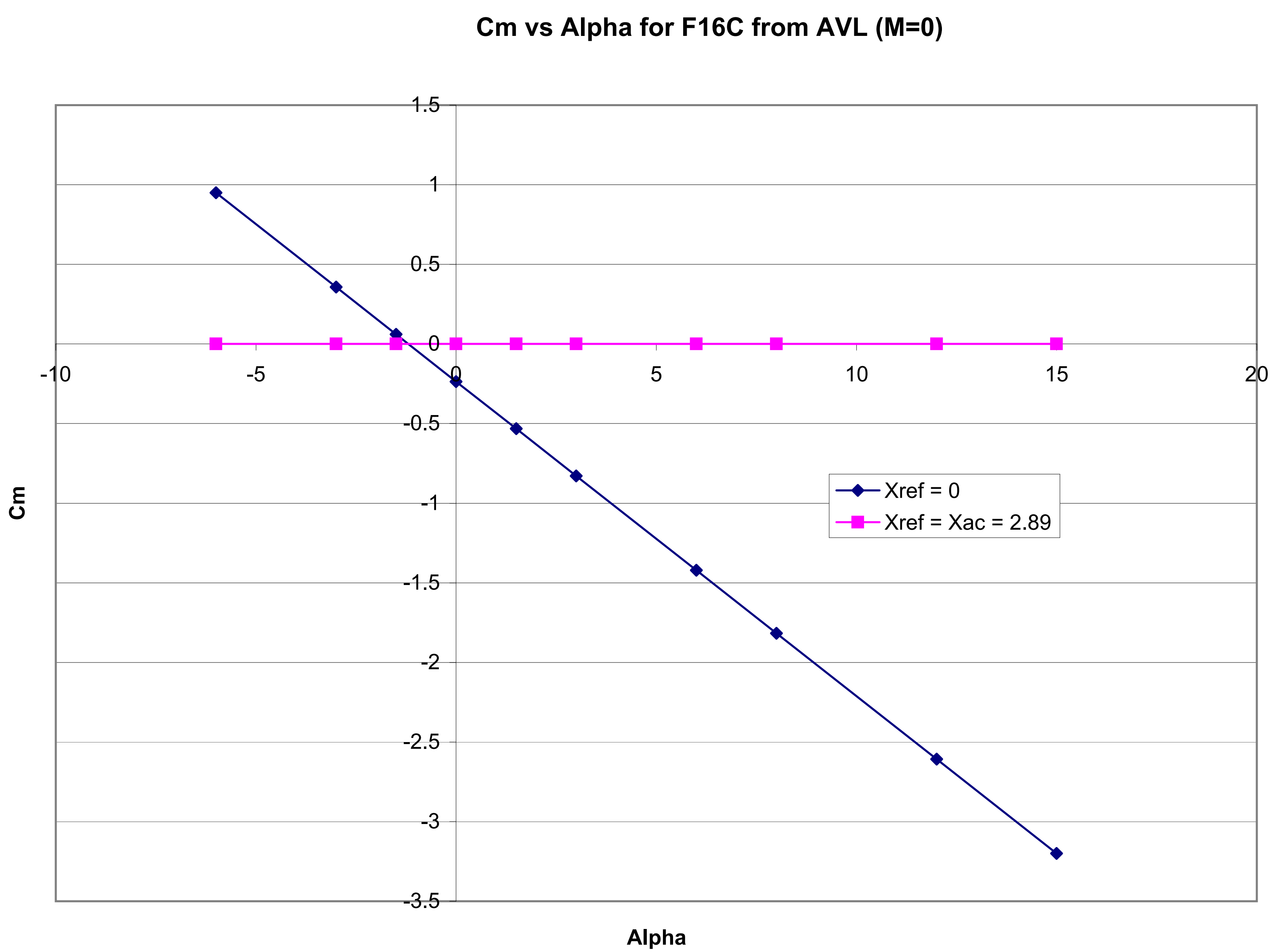
$$\Rightarrow \frac{x_{cp}}{\bar{c}} = -\frac{C_{M_{NOSE}}}{C_L}$$

So if:

$$\frac{C_{M_{NOSE}}}{C_L} = \frac{\partial C_{M_{NOSE}}}{\partial C_L},$$

this will be true. This means that

$C_M \approx 0$ at $C_L = 0$.



Cm vs Alpha for F16C from AVL (M=0)
Cm
Alpha
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-10
-5
0
5
10
15
20
Xref = 0
Xref = Xac = 2.89

Wind Tunnel Test Aerodynamic Center Characteristics for Washout and Rigid Wings (Altitude = 10,000 ft.)