16.100 Homework Assignment # 8
Due: Monday, November 14, 9am

Reading Assignment: No new reading. Relevant reading is same as from Homework #7.
Anderson, 3rd edition: Chapter 17, pages 787 – 801
Chapter 18, pages 803 – 810

Problem 1

In this problem, you will derive the 2-D integral momentum equation for boundary layers. The start of this derivation is the incompressible, 2-D boundary layer equations:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p_e}{\partial x}+\frac{\partial \tau}{\partial y} \qquad \text{(A)}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{(B)}$$

where p_e is the pressure at the edge of the boundary layer and $\tau = \mu\,\partial u/\partial y$.

a) Assuming the flow outside of the boundary layer is inviscid, irrotational flow such that Bernoulli's equation holds, show that Equation (A) can be manipulated to:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\frac{1}{\rho}\frac{\partial \tau}{\partial y} \qquad \text{(C)}$$

b) Multiply Equation (B) by $u-u_e$ and add to Equation (C). Then, integrate the resulting equation from $y=0$ to $y=\infty$ to find the boundary layer integral momentum equation:

$$\frac{\tau_w}{\rho u_e^2}=\frac{d\theta}{dx}+\left(2+H\right)\frac{\theta}{u_e}\frac{du_e}{dx} \qquad \text{(D)}$$

where $\tau_w = \mu\,\partial u/\partial y\big|_{y=0}$, θ is the momentum thickness, and $H=\delta^*/\theta$ is the shape factor.

Problem 2

In this problem, you will apply the boundary layer integral momentum equation to estimate the boundary layer properties on a flat plate (in this case $u_e = U_\infty$). To do this, we will assume that the velocity profile in the boundary layer has the following form:

$$u(x,y) = U_\infty \sin\left(\frac{\pi y}{2\delta(x)}\right) \text{ for } 0 < y < \delta(x) \text{ and } u(x,y) = U_\infty \text{ for } y > \delta(x)$$

where $\delta(x)$ is an unknown boundary layer thickness that will be determined using the integral momentum equation (Equation D).

a) Calculate δ^*, θ, H, and c_f as a function of $\delta(x)$. NOTE: in this step, simply evaluate these quantities using the assumed profile but do not use the integral momentum equation.

b) Plug the results from part a) into the integral momentum equation and derive an ordinary differential equation for $\delta(x)$ for the flow over a flat plate.

c) Solve the ordinary differential equation for $\delta(x)$ with the leading edge at $x = 0$. Plug the result into the expressions from part a). Compare these results to the Blasius' flat plate boundary solution described in the reading, specifically giving the percent error for each quantity.

Problem 3

Consider two thin airfoils moving at constant velocities near sea level altitude and at zero angle of attack. The first airfoil has a chord of 1 meters and is moving at 20 m/sec. The second airfoil has a chord of 0.5 meters and is moving at 40 m/sec. In the following questions, assume that the boundary layers of these thin airfoils can be well-approximated using Blasius' flat plate boundary layer solution.

a) Calculate and compare the drag coefficients for both airfoils. Note: account for both sides of the airfoils when you do this.

b) Calculate and compare the drag for both airfoils (again accounting for both sides of the airfoils).

c) Plot (on a single graph) and compare the skin-friction coefficient c_f versus x/c for both airfoils.

d) Plot (on a single graph) and compare the skin friction (i.e. τ_w) versus x/c for both airfoils.