16.100 Homework Assignment #2
Due:Wednesday, September 21th, 9am

Reading Assignment
Anderson, 3rd edition: Chapter 2, Sections 2.4-2.6, 2.10
Chapter 3, Sections 3.1-3.2, 3.5-3.16

Problem 1 (30%)
Useful reading: Sections 2.10, 3.6 of Anderson

The incompressible, inviscid flow equations (called the incompressible Euler equations) are:

$$\nabla \cdot \vec{V} = 0 \qquad \text{(Eq.1)}$$

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p \qquad \text{(Eq. 2)}$$

a) Starting from the incompressible Euler equations, derive the following 'Bernoulli-like' equation:

$$\rho \frac{\partial \vec{V}}{\partial t} + \nabla\left(p + \tfrac{1}{2}\rho\left|\vec{V}\right|^2 \right) = \rho \vec{V} \times \vec{\omega}$$

where $\vec{\omega} = \nabla \times \vec{V}$ is the vorticity. The following vector calculus identity might be helpful:

$$\left(\vec{V} \cdot \nabla\right)\vec{V} = \nabla\left(\tfrac{1}{2}\left|\vec{V}\right|^2 \right) - \vec{V} \times \vec{\omega}$$

b) Show that the total pressure, $p + \tfrac{1}{2}\rho\left|\vec{V}\right|^2$, is constant along a streamline in a steady, inviscid flow.

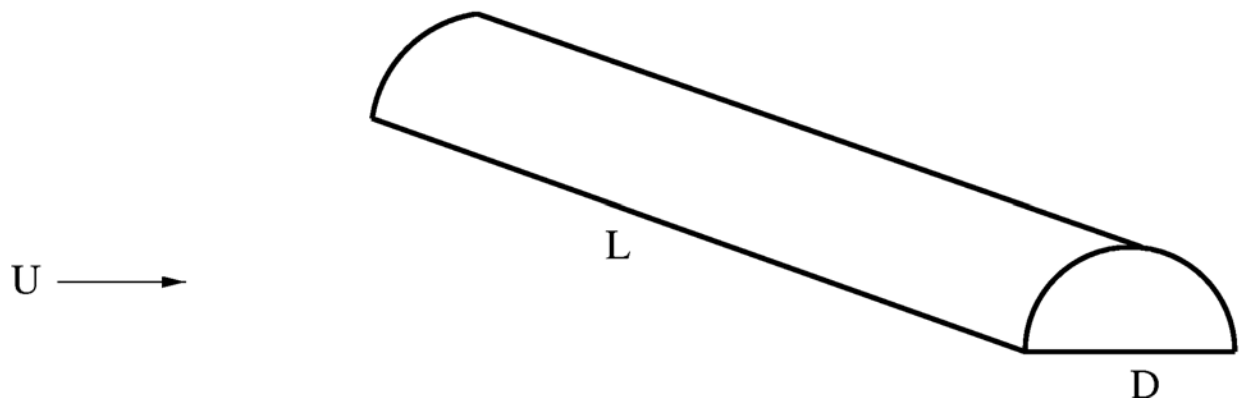
c) Show that the total pressure is constant everywhere in a steady, inviscid, and irrotational flow.

d) By taking the curl of the incompressible, inviscid momentum equation (Eq. 2 above), show that:

$$\frac{D\vec{\omega}}{Dt} = \vec{\omega} \cdot \nabla \vec{V}$$

e) If the vorticity of a fluid element is $\vec{\omega}_0$ at time $t = 0$ and the flow is two-dimensional, prove that the vorticity is always equal to $\vec{\omega}_0$ for any $t > 0$.

Problem 2 (40%)
Useful reading: Section 3.13 of Anderson



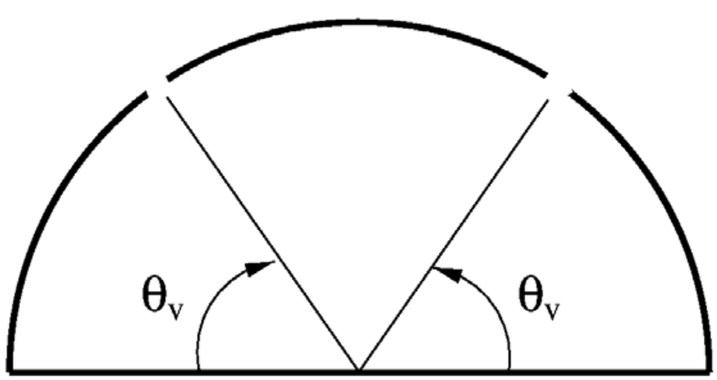
Consider the semi-cylinder aircraft hangar shown above. Assume:

- Far upstream of the hangar, the wind has uniform speed U and is perpendicular to the hangar length. The upstream pressure is p_∞.
- The end effects are small.
- The flow is inviscid to good approximation.

Answer the following questions:

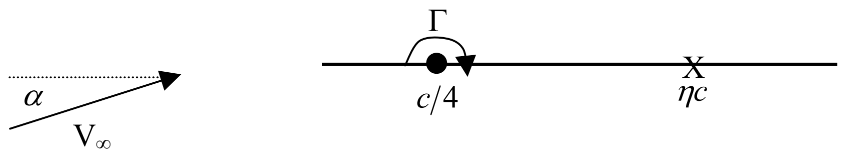
a) Assume the pressure in the interior of the hangar is p_H. If the circular roof can withstand a maximum net vertical force of $F_{\max}$, what is the maximum velocity the hangar can withstand?

b) To reduce the pressure differential between the inside and outside of the hangar, vents are to be placed at positions θ_v as shown in the figure below. When this is done, the pressure inside the hangar will be equal to the pressure outside the hangar at the vent location. What should θ_v be to make the net vertical force on the roof zero?



Problem 3 (30%)
Useful reading: Section 3.7, 3.14, 3.16 of Anderson

A simple model for a thin, symmetric airfoil (i.e. no camber) is to place a point vortex at the quarter-chord location and then satisfy flow tangency at a selected control point located one the chord line at ηc as shown in the figure below.



Thin airfoil theory gives that the lift coefficient for a symmetric thin airfoil at small angle of attack is given by $c_l = 2\pi\alpha$. Find the location of the control point (i.e. find the value of η) such that the lift coefficient estimated from the simple point vortex model is identical to the thin airfoil results.