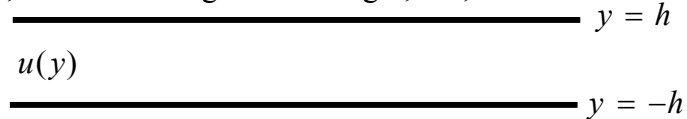**16.100 Homework Assignment # 1**
Fall 2005
Due: Monday, September 12, 9am

Reading Assignment
Anderson: Section 2.1 – 2.3; Section 2.9-2.13, Section 15.3

Problem 1

In this problem, consider a long duct of height, $2h$, as shown below:

$y = h$

$u(y)$

$y = -h$

Three velocity fields which can model the flow in the ducts are:
- Uniform flow: $u(y) = U_\infty$

- Couette flow: $u(y) = \dfrac{U_w}{2}\left(1 + \dfrac{y}{h}\right)$

- Poiseuille flow: $u(y) = U_0\left[1 - \left(\dfrac{y}{h}\right)^2\right]$

We assume that the other velocity components are zero. The uniform flow is representative of very high Reynolds number (though without the boundary layers). Couette flow is the (laminar) flow that results when the upper wall is moving at velocity U_w and the bottom wall is stationary. Poiseuille flow is the (laminar) flow that results when a pressure drop is applied from end-to-end.

a) For all of the velocity distributions, sketch the streamlines.

b) For all of the velocity distributions, calculate the vorticity.

c) For all of the velocity distributions, state whether the flows are rotational or irrotational.

d) For all of the velocity distributions, calculate the strain rates.

e) For all of the velocity distributions, calculate the acceleration, $D\vec{u}/Dt$.

f) Consider a slice of the fluid between $y = \dfrac{h}{4}$ and $y = \dfrac{h}{2}$. If the dynamic viscosity of the fluid is μ, determine the total force per unit length (and per unit depth into the drawing) exerted on the slice by the viscous stresses for all of the velocity distributions.

Problem 2

In this problem, we will look at the velocity behavior of vortices. We will assume that the velocity in these vortices is zero in the radial and axial directions, and in the circumferential direction only depends on the radial location, i.e.

$$u_r = 0 \qquad u_\theta = u_\theta(r) \qquad u_z = 0.$$

a) For this type of velocity distribution, prove that the only component of vorticity which is possibly non-zero is the axial component, ω_z.

b) The only portion of the shear strain rate that could be non-zero for this velocity field is

$\varepsilon_{r\theta} = \dfrac{\partial u_\theta}{\partial r} - \dfrac{u_\theta}{r}$ (you do not need to prove this). Derive the $u_\theta(r)$ for which $\varepsilon_{r\theta} = 0$. There will be a single free parameter (i.e. a constant) remaining in this zero-strain vortex. What is the physical meaning of this parameter? For this zero-strain vortex, calculate the vorticity.

c) Now derive the $u_\theta(r)$ for which $\omega_z = 0$. As in the previous case, there will be a single free parameter remaining in this zero-vorticity (i.e. irrotational) vortex. What is the physical meaning of this parameter? For this irrotational vortex, calculate the strain rate.

d) For the zero-strain velocity field from part b, calculate the acceleration, $D\bar{u}/Dt$ in cylindrical coordinates. To do this, first transform the velocity vector from a cylindrical coordinate system to Cartesian coordinate system. Then, apply the Cartesian coordinate system formula for the acceleration $D\bar{u}/Dt$. Finally, transform the acceleration vector back to cylindrical coordinates.

e) The trailing, wing-tip vortices behind high-aspect ratio wings, such as the wings of commercial aircraft, have a velocity field which is often well-approximated by,

$$u_\theta(r) = \frac{\Gamma_\infty}{2\pi r}\left[1 - e^{-(r/\delta)^2}\right],$$

where the constants Γ_∞ and δ are the vortex circulation (or strength) and vortex core size, respectively. Calculate the vorticity and the strain rate for this vortex. How is this vortex related to the vortices derived in parts (b) and (c)? Slightly downstream of the aircraft, the core of the vortex is typically found to be about one-fourth of the wing span, i.e. b/4. Also, using the results of lifting line theory, the strength of the vortex can be related to the lift being generated, specifically, $L = \dfrac{\pi}{4}\rho_\infty V_\infty \Gamma_\infty b$. For a Boeing 747-400 series aircraft at take-off carrying its maximum load, estimate the maximum swirl velocity in the wing tip vortices.