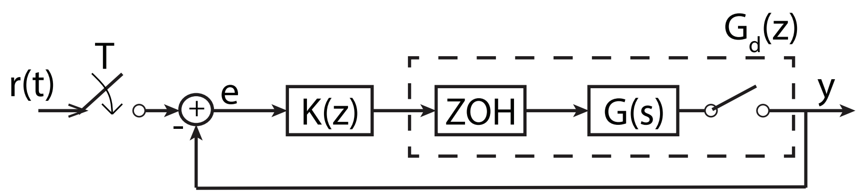
16.06 Principles of Automatic Control
Recitation 13



$$\frac{E}{R} = \frac{1}{1+KG} \rightarrow E = \frac{E}{R} \cdot R$$

$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s)$, in continuous time.

$e_{ss} = \lim_{k\to\infty} e[k] = \lim_{z\to 1}(z-1)E(z)$

For a step input, $r = \sigma(t)$.

C	D
$e_{ss} = \frac{1}{1+K(0)\,G(0)}$	$e_{ss} = \frac{1}{1+K_d(1)G_d(1)}$
$K_p = K(0)G(0)$	$K_p = K_d(1)G_d(1)$

Given a type I system:

$$L(z) = G_d(z)\,K_d(z) = \frac{L_0(z)}{z-1}$$

Find K_v :

$$\frac{E(z)}{R(z)} = \frac{1}{1+K_d(z)G_d(z)}$$

$$E(z) = \frac{E(z)}{R(z)} \cdot R(z) = \frac{1}{1+\frac{L_0(z)}{z-1}} \cdot \frac{T_z}{(z-1)^2}$$

where T_z is unit ramp from the table.

Therefore,

$$\begin{aligned} e_{ss} &= \lim_{z\to 1}(z-1)\frac{1}{1+\frac{L_0(z)}{z-1}}\cdot\frac{T_z}{(z-1)^2} \\ &= \lim_{z\to 1}\frac{T_z}{z-1+L_o(z)} \\ &= \frac{T}{L_0(1)} \\ &= \frac{1}{K_v} \end{aligned}$$

$$\boxed{\Rightarrow\ K_v = \frac{L_o(1)}{T}, \text{ in discrete time.}}$$

Using the w-transform:

$$\text{Take } \frac{L_0(z)}{z-1} = \frac{L_0\left(\frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}\right)}{\frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}-1} = \frac{L_0\left(\frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}\right)\left(1-\frac{wT}{2}\right)}{wT}$$

$$\begin{aligned} \therefore\ K_v &= \lim_{w\to 0}\frac{L_0\left(\frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}\right)\left(1-\frac{wT}{2}\right)}{wT}\cdot w \\ &= \frac{L_0(1)}{T} \end{aligned}$$

$$\boxed{\text{The w-transform preserves } K_p \text{ and } K_v.}$$

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Fall 2012