

$$a) I_{sp} = \frac{u_e}{g} = \frac{1}{g} \left[ M_e \sqrt{\gamma R T_c \left( \frac{1}{1 + \frac{\gamma-1}{2} M_e^2} \right)} \right]$$

NEED TO ITERATE TO FIND  $M_e$  FOR GIVEN  $\frac{A^*}{A_e}$

$$\frac{A_e}{A^*} = \frac{0.01}{0.0006} = 16.67 = \frac{1}{M_e} \left[ \frac{1 + \frac{\gamma-1}{2} M_e^2}{\frac{\gamma+1}{2}} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \Rightarrow M_e = 4.27$$

$$\therefore I_{sp} = 271 \text{ s} \quad (u_e = 2654 \text{ m/s})$$

$$T = \dot{m} u_e + A_e (p_e - p_o) \quad p_o = \text{ATMOSPHERIC PRESSURE AT LAUNCH}$$

$$\dot{m} = \sqrt{\gamma} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{p_c}{\sqrt{R T_c}} A^* = 1.85 \text{ kg/s}$$

$$p_e = \frac{p_c}{\left[ 1 + \frac{\gamma-1}{2} M_e^2 \right]^{\frac{\gamma}{\gamma-1}}} = 20.0 \text{ kPa}$$

$$\therefore T = 4102 \text{ N (AT LAUNCH)} \quad [= 4915 \text{ N if you neglect pressure term}]$$

$$b) \text{ AT BURNOUT} \quad u_{bo} = g \left[ I_{sp} \ln \left( \frac{m_{\text{initial}}}{m_{\text{final}}} \right) - t_{bo} \right]$$

$$h_{bo} = g \left[ -t_{bo} I_{sp} \frac{\ln \left( \frac{m_i}{m_f} \right)}{\left( \frac{m_i}{m_f} - 1 \right)} + t_{bo} I_{sp} - \frac{1}{2} t_{bo}^2 \right]$$

$$t_{bo} = m_{\text{propellant}} / \dot{m} = 35 \text{ kg} / 2.4 \text{ kg/s} = 18.9 \text{ s}$$

$$\text{SO} \quad u_{bo} = 3010 \text{ m/s} \quad h_{bo} = 22532 \text{ m}$$

$$\text{GLIDE PHASE} \quad \frac{1}{2} m_{\text{final}} u_{\text{final}}^2 = m_{\text{final}} g \Delta h$$

$$\Delta h = 462370 \text{ m}$$

$$\text{TOTAL HEIGHT} = h_{bo} + \Delta h = \boxed{485 \text{ km}}$$