"The Language of Engineering"

Engineers work in the real world. They need a language to describe that world - to quantify it and to model it. The language of Engineering is largely based on Mathematics (8.01, 02, 03) but there are a few other important concepts that are common to several branches of engineering. We are going to teach you these as "Unified Lectures". This should include a fair amount of material from 8.01

Dimensions and Units

- A dimension is a physical quantity which can be measured directly.
- A collection of dimensions are used as the basis for a system of units
- Units quantify dimensions We will use two systems of units: SI (metric) and British System (now a misnomer)

See inside cover of Crandall, Dahl, & Lardner

DIMENSIONS (Fundamental in SI)		UNITS	
Length	[L]	m	Also(nm, mm, mn, km…)
Time	[T]	s	(not Sec's - except in S&S!)
Mass	[M]	Kg	(g)
Current	[C]	A	Ampere
Temperature	[θ]	K	(not ^{o} centigrade)
Luminous Intensity	(I)	Cd	(Candela)
Amount	(n)	M	Mole

Dimensions of other quantities are derived:

Example: Newton's Law

$$F = Ma \qquad \left[M\frac{L}{T^2}\right]$$

Force is a derived unit in SI

In British System force is a fundamental unit:

Dimension	SI (Unit)	English Unit	Conversion
Length	m	ft. (in)	0.305/ft
Time	s	Sec	-
Mass	Kg	Slug	14.6 Kg/slug
Force	N	Lb.	4.45N/lb.
Pressure	Pa (Pascal)	psi	6900 Pa/Psi

1 Newton = 1Kg m/s^2
1 pound = 1 Slug ft/S^2
1 Pascal = 1 N/m^2
1 Psi = 1 lb/in^2

} Same Dimensions, Different Units

Be VERY careful to give Units.
Be VERY careful not to mix units. "cannot combine or compare apples & oranges"
(You many know this but the Mars Climate Orbiter Team did not!)

Be very careful to use consistent magnitudes.
Time in s not hrs. Length in $km \rightarrow m \times 10^3$

Use dimensions as a check.

$$E = MC^?$$

energy — mass — speed of light

$$ML^2T^{-2} = M\left[LT^{-1}\right]^?$$ Clearly ? = 2

This is the start of <u>Dimensional Analysis</u>

Which can allow us to identify the controlling physical quantities in unfamiliar or complicated quantities

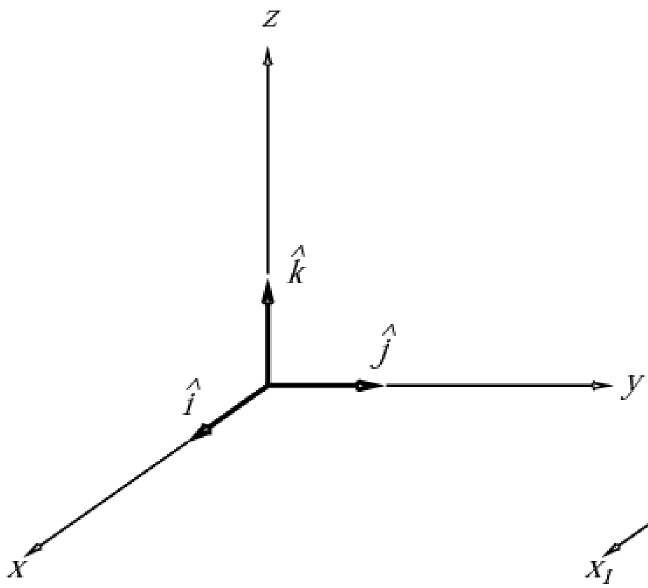
Fluid Dynamics and Thermodynamics make extensive use of dimensional analysis.

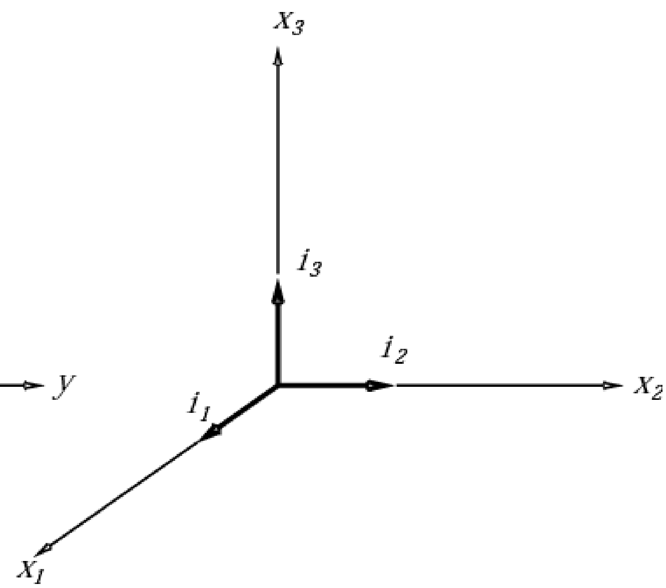
Coordinate Systems

Dimensions and units allow us to quantify magnitudes (Scalar quantities).

But engineers need to quantify <u>vector</u> quantities (e.g. force, velocity, acceleration, etc.) - Need a coordinate system.

Most commonly we will use a Cartesian (rectangular) coordinate system. (from Descartes). The axes are commonly labeled x, y, z. An alternative and frequently useful notation uses subscripts: x_1, x_2, x_3.





$\hat{i}, \hat{j}, \hat{k}$ are unit vectors along x, y, z

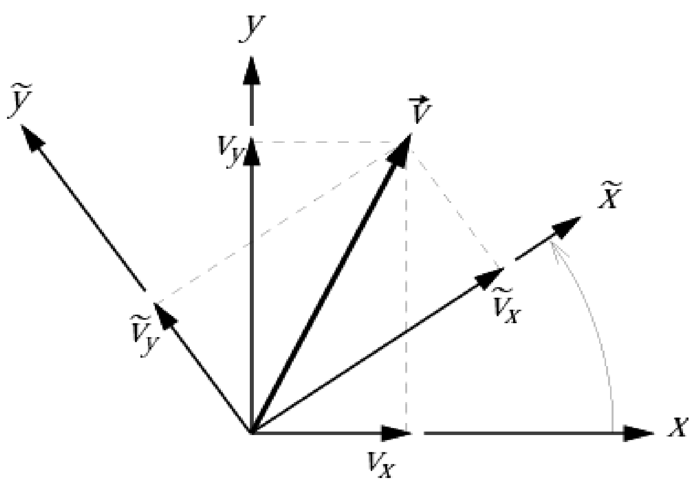
$\underline{i}_1, \underline{i}_2, \underline{i}_3$ are unit vectors along x_1, x_2, x_3

Other common notations for unit vectors are $\hat{e}_x\ \hat{e}_y\ \hat{e}_z$ and $\hat{e}_1\ \hat{e}_2\ \hat{e}_3$

A right-handed coordinate system has $\hat{i} \times \hat{j} = \hat{k}$. This is the standard convention.

(A left-handed system is the mirror image, and has $\hat{i} \times \hat{j} = -\hat{k}$. Not recommended).

A vector is described by its components along some chosen axes. Different axis choices will produce different components for the same vector. Example…



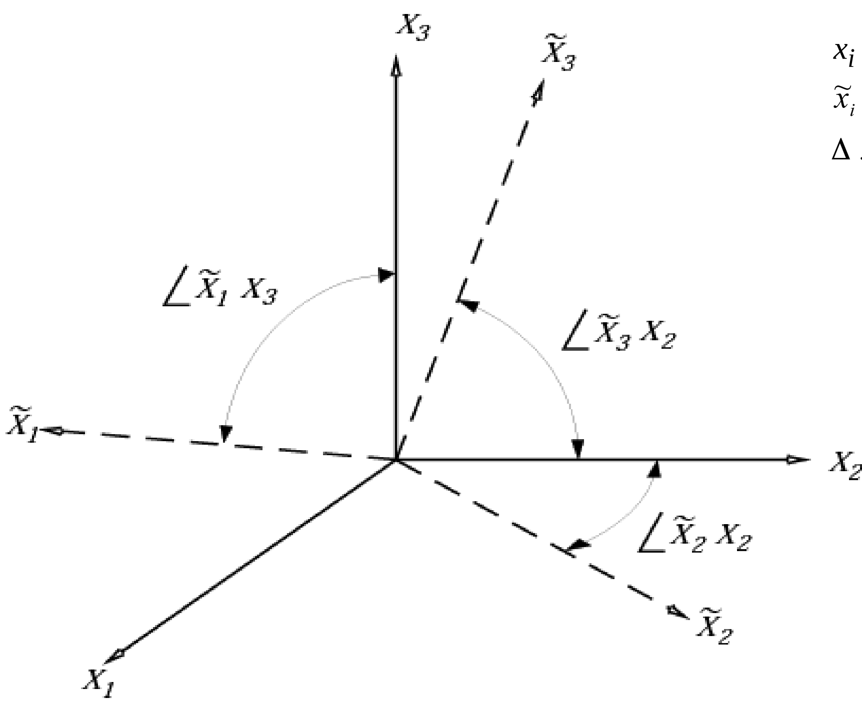
(v_x, v_y) and $(v_{\tilde{x}}, v_{\tilde{y}})$ are alternative ways to describe the same vector $\vec{v}$

IMPORTANT CONCEPT
The physical quantity does not change because it is described in a different coordinate system (or different units). It remains the same, only the description changes.

Transformation of Coordinate Frames

Often we need to change between different coordinate systems, e.g.,
ground/aircraft
Observer/radar station
Docking spacecraft

Translation of coordinates is straightforward
Rotation needs some explanation



x_i = original axes
$\tilde{x}_i$ = rotated axes
$\Delta\, x_m x_{\tilde{x}}$ = angle between
x_m and $x_{\tilde{n}}$ axes

Invoke direction cosines $l_{m\tilde{n}} = Cos\Delta x_m x_{\tilde{n}}$

$$x_1 = l_{1\tilde{1}}\tilde{x}_1 + l_{1\tilde{2}}\tilde{x}_2 + l_{1\tilde{3}}\tilde{x}_3$$

Effectively projects components of $\tilde{x}_m$ onto x_1

Note: because *Cos* is an even function,
$l_{m\tilde{n}} = l_{\tilde{n}m}$
but
$l_{\tilde{m}n} \neq l_{\tilde{n}m}$

Note: $l_{m\tilde{n}}$ can be defined using the Scalar product operation for vectors: $l_{m\tilde{n}} = \underline{i}_m \cdot \underline{i}_{\tilde{n}}$

Similarly for x_2 and x_3

$$x_2 = l_{2\tilde{1}}\, \tilde{x}_1 + l_{2\tilde{2}}\, \tilde{x}_2 + l_{2\tilde{3}}\, \tilde{x}_3$$

$$x_3 = l_{3\tilde{1}}\, \tilde{x}_1 + l_{3\tilde{2}}\, \tilde{x}_2 + l_{3\tilde{3}}\, \tilde{x}_3$$

and vice versa

e.g.

$$\tilde{x}_1 = l_{\tilde{1}1}\, x_1 + l_{\tilde{1}2}\, x_2 + l_{\tilde{1}3}\, x_3$$

Same applies to transformation of any vector quantity defined in x_n and $\tilde{x}_n$ (velocities, forces, etc.)

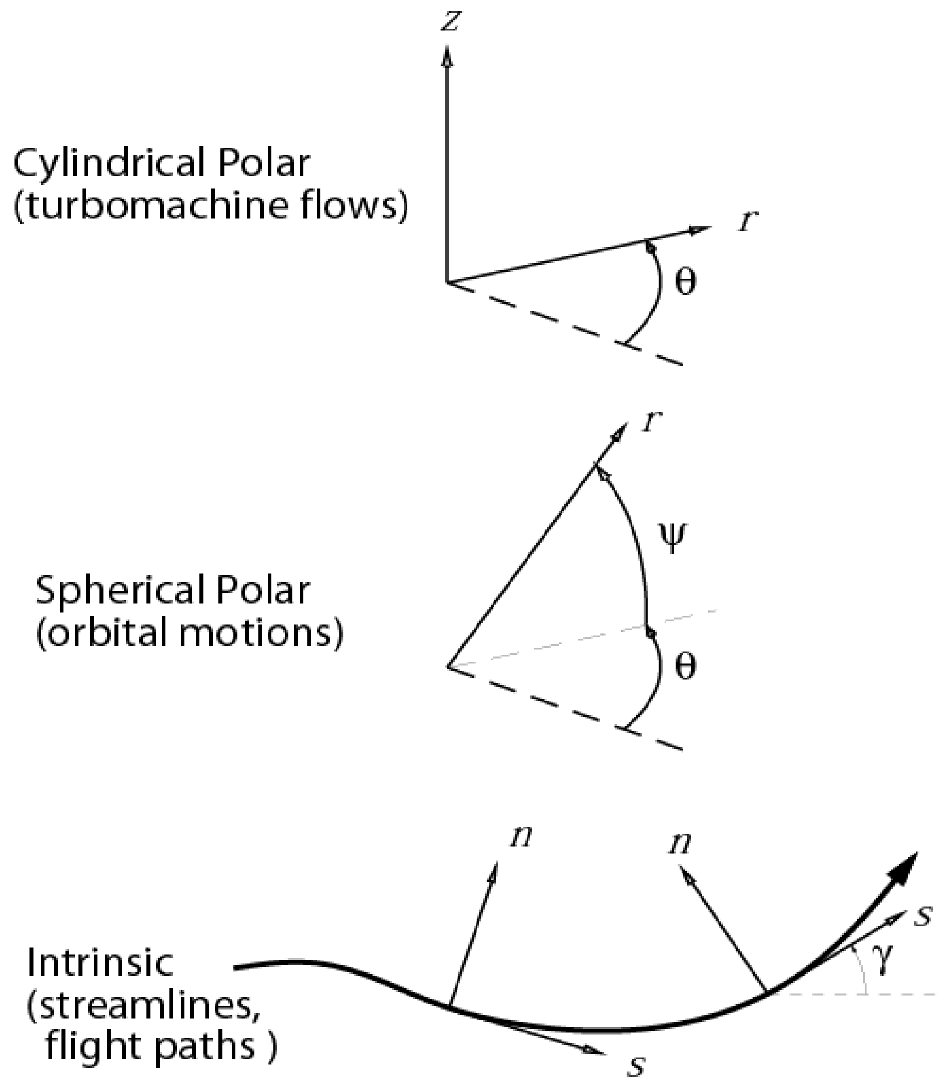
Effectively, we have defined a transformation matrix $\bar{l}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \underbrace{\begin{pmatrix} l_{1\tilde{1}} & l_{1\tilde{2}} & l_{1\tilde{3}} \\ l_{2\tilde{1}} & l_{2\tilde{2}} & l_{2\tilde{3}} \\ l_{3\tilde{1}} & l_{3\tilde{2}} & l_{3\tilde{3}} \end{pmatrix}}_{\bar{l}} \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{pmatrix}$$

REPEAT: Even though we are transforming the coordinates the <u>physical quantities remains unchanged.</u> (Physical vector as opposed to <u>mathematical</u> description of it.)

<u>Other Coordinate Systems</u> are used when it makes sense to describe a body/system in that manner.

Examples of different coordinate systems:



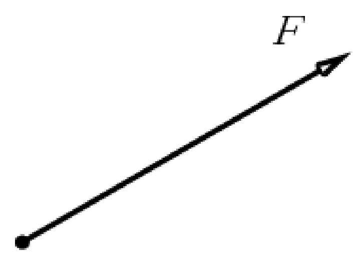
Vector Mechanics

Forces

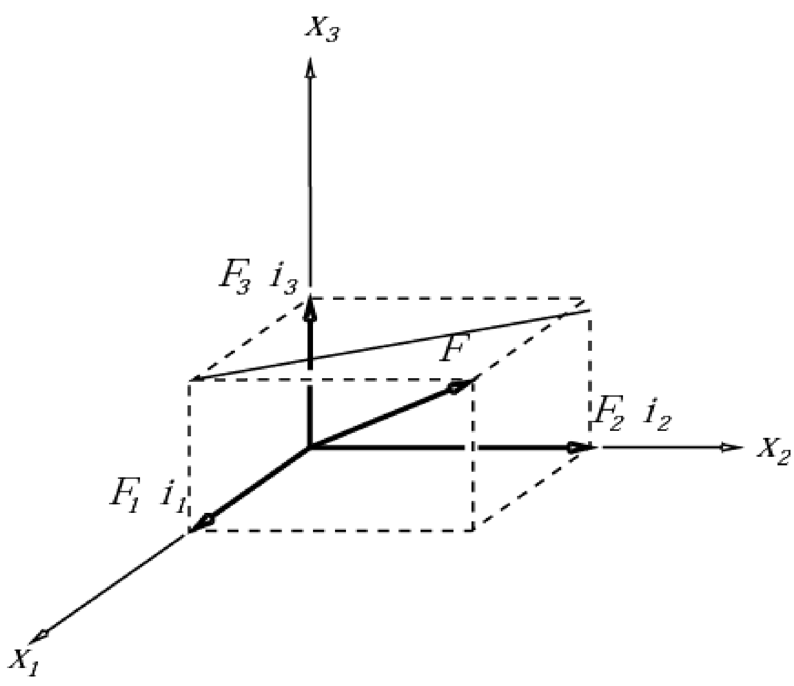
Definition: A force is a measure of the action of one body or medium on another (push or pull).

Force has
- magnitude
- direction
- point of application

and are best represented by vectors

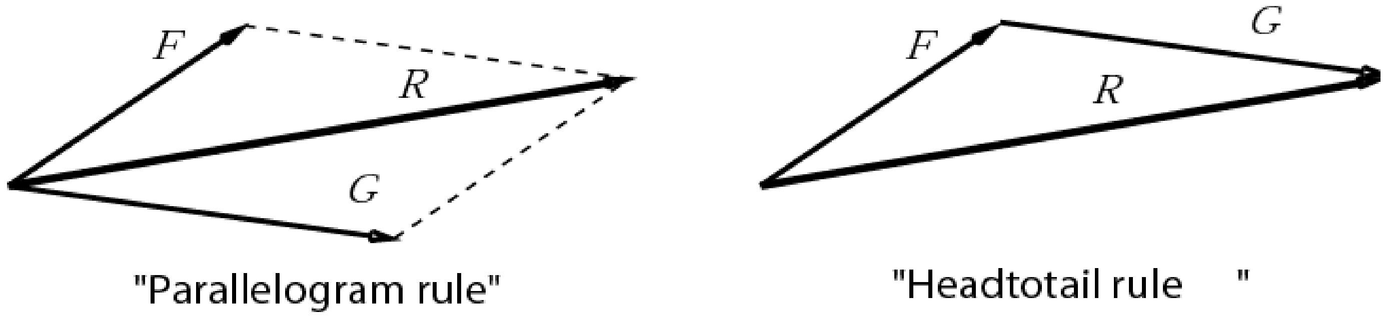


The force can be expanded as: $\underline{F} = F_1\,\underline{i}_1 + F_2\,\underline{i}_2 + F_3\,\underline{i}_3$



where $F_i, (F_1, F_2, F_3)$ are the three components of the force in each of the three associated directions.

Forces are vectors, sum by vector addition



With either rule, we write

$\underline{R} = \underline{F} + \underline{G}$

$\underline{R} = (F_1 + G_1)\underline{i}_1 + (F_2 + G_2)\underline{i}_2 + (F_3 + G_3)\underline{i}_3$

so adding vectors is done by simply adding their components.

This can also be written as: $R = \sum_1^3 R_m i_m$

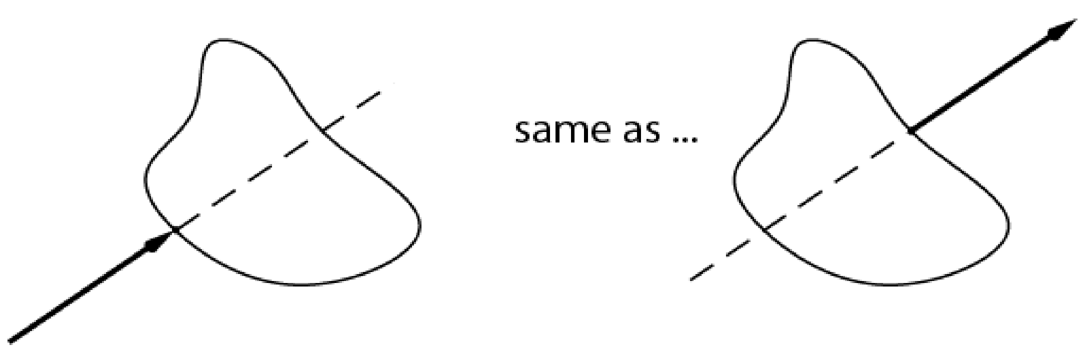
Which will lead us into indicial or tensor notation

--to be covered in materials and structures lectures.

There are two important force relationships:

Transmissibility of Forces

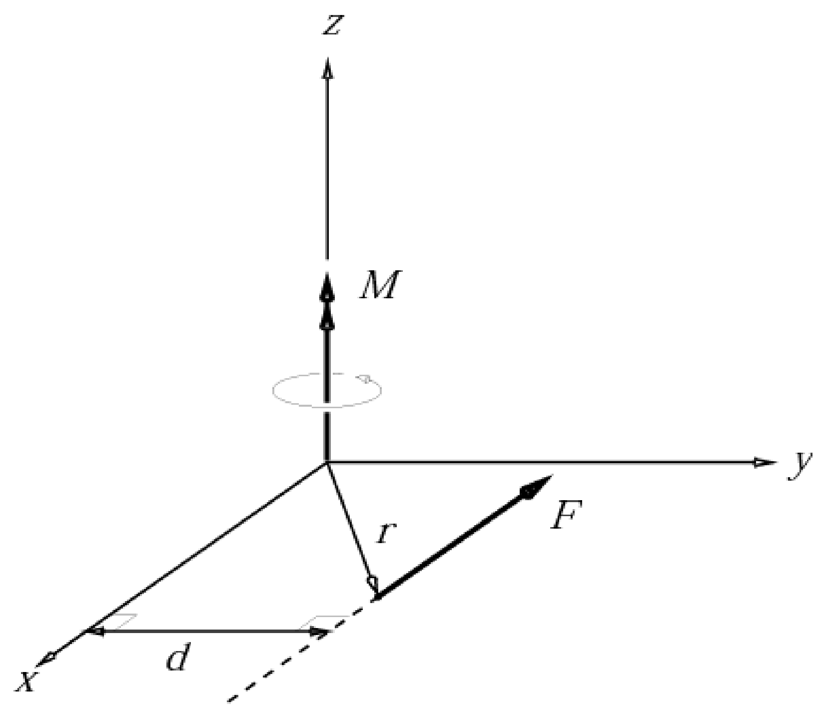
In addition to a magnitude, direction and point of application, forces also have a line of action. For a rigid body, a force can be applied anywhere along a given line of action for the same effect.



Moments

Definition: A moment is a force acting about (around) an axis.

Example:



Force $\underline{F}$ is parallel to x in $x-y$ plane. d is perpendicular distance between line of action of force and axis about which it is acting. Magnitude of moment about z is:

$|\underline{M}| = |\underline{F}|.d = Fd$

direction of moment is given by right hand rule

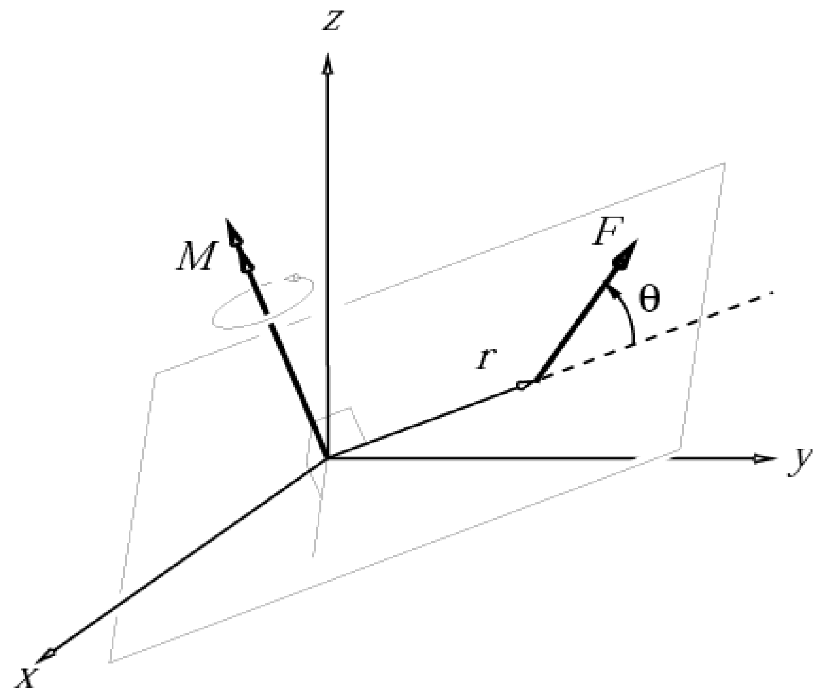
(moment arm vector) x (force vector) = (moment vector)

In general, the moment of $\underline{F}$ about the origin is:

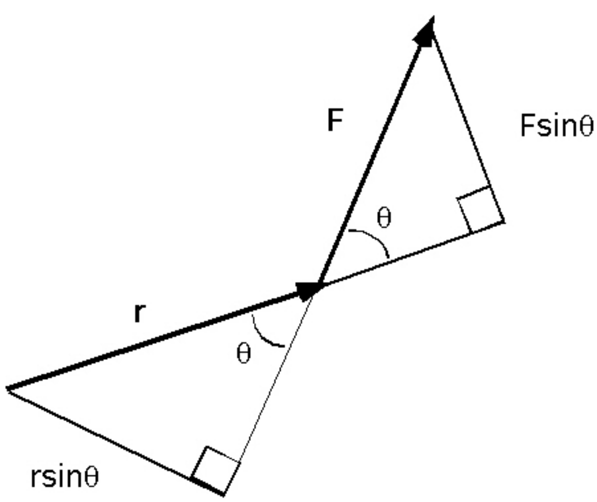
$\underline{M} = \underline{r} \times \underline{F}$ (vector cross product)

$\underline{r}$ = vector from origin to point of application of $\underline{F}$ (position vector)

General Case: Consider the plane containing both $\underline{r}$ and F .



Overhead view of plane:



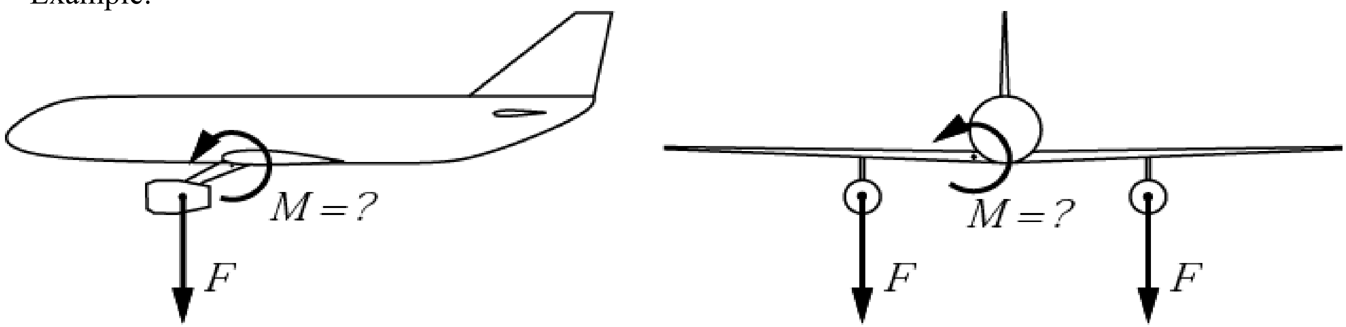
As before, the magnitude of moment is magnitude of force times perpendicular distance from origin to line of action of force.

$$|\underline{M}| = |\underline{r}|\,|\underline{F}|\ Sin\theta$$

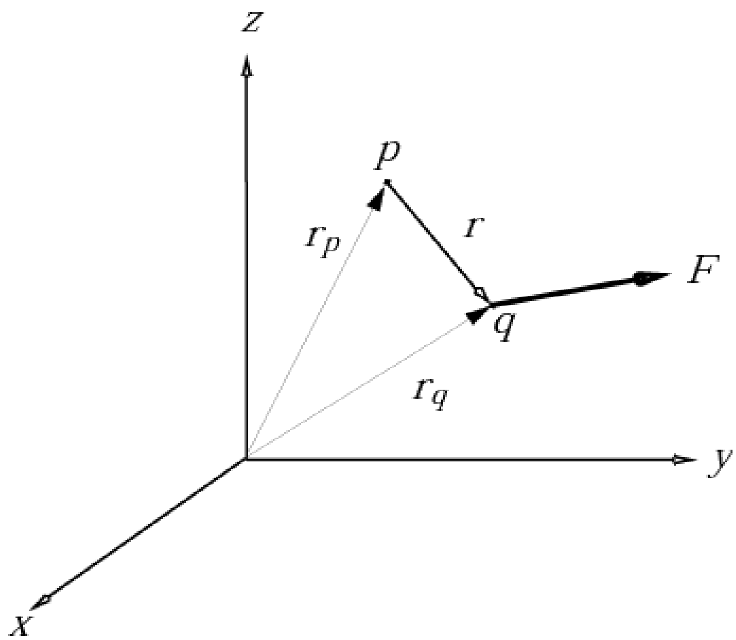
The moment vector direction is perpendicular to both $\underline{r}$ and F , and hence perpendicular to the plane containing r and F .

What if we want to know the moment about some arbitrary point?

Example:



Engine weight exerts moment about pylon root, about wing root, or about any other point of interest.



$\underline{r}_p$ and $\underline{r}_q$ are position vectors from origin to points p and q respectively.
$\underline{r}$ is vector from p to q

So: $\underline{r}_q = \underline{r}_p + \underline{r}$, or $\underline{r} = \underline{r}_q - \underline{r}_p$

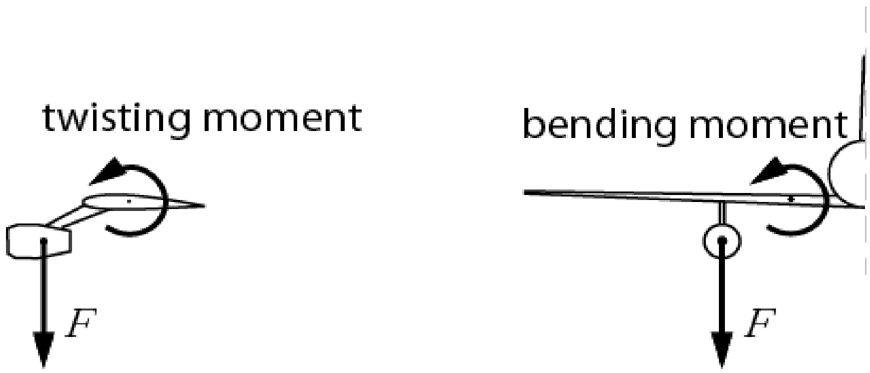
Moment of $\underline{F}$ about $\underline{p}$ is:

$$\underline{M} = \underline{r} \times \underline{F} = \left(\underline{r}_q - \underline{r}_P\right) \times \underline{F}$$

Same idea!

Often in structures we want to understand the effect of moments about parts of the structure.

For instance:



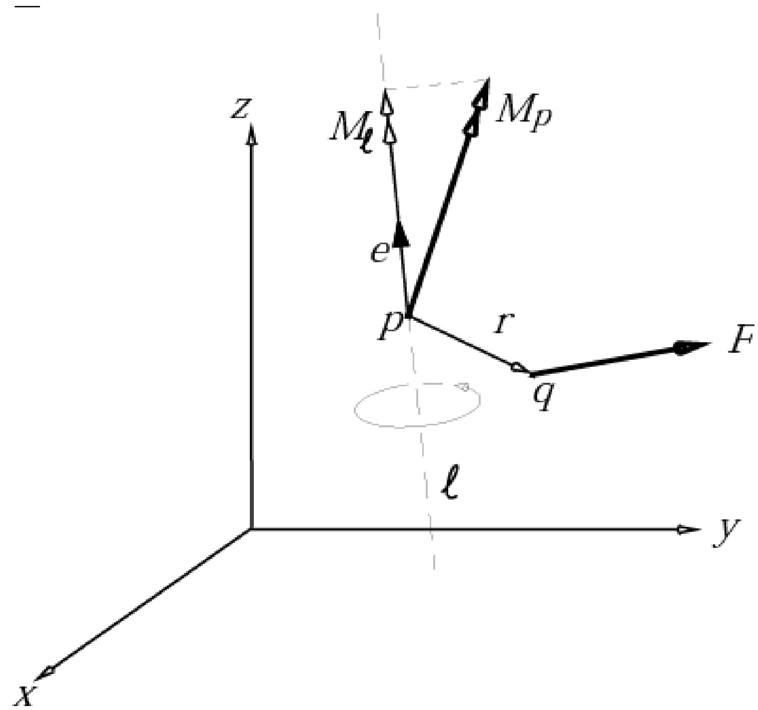
Moment due to engine weight twists wing about spanwise axis, but bends wing about chordwise axis.

<u>Moment about Arbitrary Axis</u>

Consider force $\underline{F}$ at point q and we want to know moment about line `l`

Define <u>any</u> point p on `l`, and a unit vector $\underline{e}$ from point p along `l`

We know that $\underline{M}_p = \underline{r} \times \underline{F}$



So component of $\underline{M}_p$ along l

Can be found via dot product (scalar product) $M_l = e.(rxF)$. M_l is a scalar, but can be given a direction (made into a vector) using e :

$$M_l = M_l e = [e.(rxF)]e$$

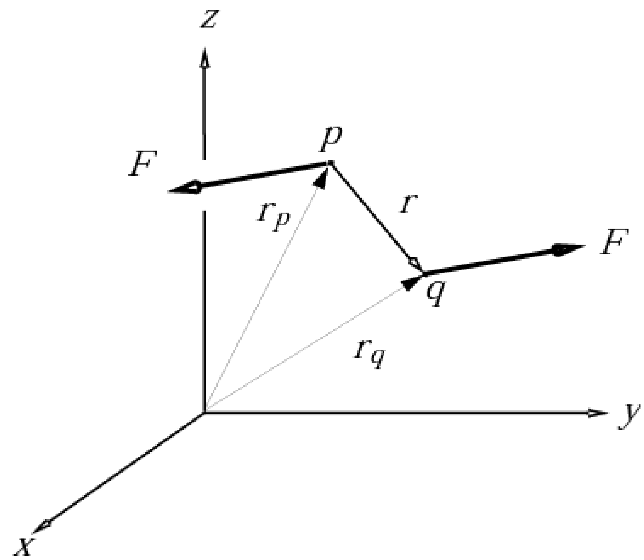
Physically, M_l is the component of M_p which is "twisting" about l.

NOTE: There is also a component of $\underline{M}$ which is perpendicular to l

(What describes this effect?) - bending

Couple - (or pure moment)

Results from 2 parallel coplanar forces of equal magnitude and opposite directions.



$\underline{F}_p$ and $\underline{F}_q$ are equal magnitude but opposite directions.

Thus: Net force $\underline{F} + (-\underline{F}) = 0$ - none

Net Moment (about origin)

$$\underline{M} = \underline{r}_p \times (-\underline{F}) + r_q \times \underline{F}$$

$$(\underline{r}_q - \underline{r}_p) \times \underline{F} = \underline{r} \times \underline{F}$$

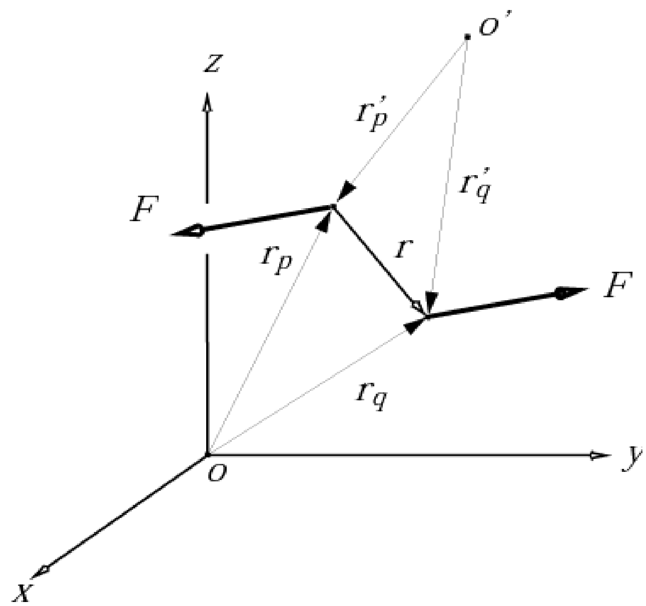
Vector product is associative

$\underline{C} = \underline{r} \times \underline{F}$

Couple

NOTE: Magnitude (and direction) of couple <u>does not change</u> as point about which couple is taken changes.

Why? Moment arm, $\underline{r}$ stays the same no matter which origin is used for r_p and r_q

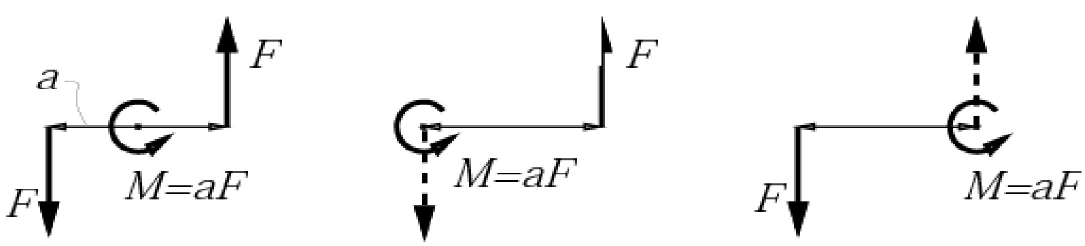


$$\underline{C}' = \underline{r}'_p \times (-\underline{F}) + \underline{r}'_p \times \underline{F}$$
$$= (\underline{r}'_p - \underline{r}'_p) \times \underline{F}$$
$$= \underline{r} \times \underline{F} \quad \text{as before}$$

product is associative

In 2-D

$$M = C = \frac{2aF}{2} = aF$$



NOTE: Pure moments can "move around" anywhere since they have the same effect about any point.