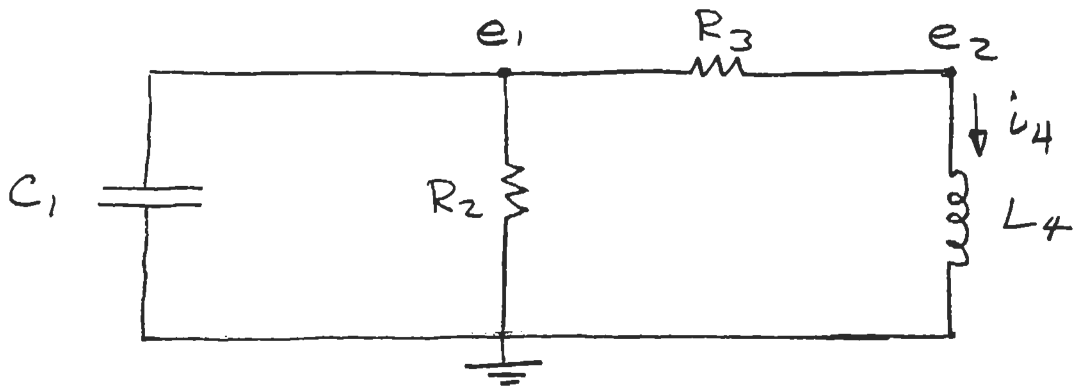
Lecture S9

RLC Circuits

Consider the RLC Circuit below:



$C_1 = 1/2\ F \quad R_2 = 4\,\Omega \quad R_3 = 6\,\Omega \quad L_4 = 2\,H$

Try to solve using node method:

$$e_1: \quad \underbrace{C_1 \frac{d}{dt}(e_1 - 0)}_{\substack{\text{current out of} \\ e_1 \text{ through } C_1}} + \underbrace{\frac{1}{R_2}(e_1 - 0)}_{\substack{\text{current out of} \\ e_1 \text{ through } R_2}} + \underbrace{\frac{1}{R_3}(e_1 - e_2)}_{\substack{\text{current out} \\ \text{of } e_1 \text{ through} \\ R_3}} = 0$$

$$\Rightarrow \quad C_1 \dot{e}_1 + (G_2 + G_3) e_1 - G_3 e_2 = 0$$

$$e_2: \quad \underbrace{\frac{e_2 - e_1}{R_4}}_{\substack{\text{current out of } e_2 \\ \text{through } R_4}} + \underbrace{i_4}_{\substack{\text{current out} \\ \text{of } e_2 \text{ through } L_4}} = 0$$

What is i_4?

Constitutive law:

$$v_4 = L_4 \frac{di_4}{dt}$$

$$\Rightarrow \frac{di_4}{dt} = \frac{1}{L_4}(e_2 - 0) = \frac{1}{L_4} e_2$$

Integrating,

$$i_4(t) = \frac{1}{L_4} \int_0^t e_2(\tau)\, d\tau + i_4(0)$$

What a pain! We could use this, but it's easier to work with differential form:

$$e_1: \quad C_1 \dot{e}_1 + (G_2 + G_3) e_1 - G_3 e_2 = 0$$

$$e_2: \quad -G_3 e_1 + G_3 e_2 + i_4 = 0$$

$$L_4: \quad \frac{1}{L_4} e_2 - \frac{di_4}{dt} = 0$$

So we have 3 equations, 3 unknowns

Solve just like before. Assume

$$e_1(t) = E_1 e^{st}$$

$$e_2(t) = E_2 e^{st}$$

$$i_4(t) = I_4 e^{st}$$

Plug into equations:

$$e_1: \quad (C_1 s + G_2 + G_3) E_1 - G_3 E_2 = 0$$

$$e_2: \quad -G_3 E_1 + G_3 E_2 + I_4 = 0$$

$$L_4: \quad \frac{1}{L_4} E_2 - s I_4 = 0$$

Eliminate I_4:

$$I_4 = \frac{1}{L_4 s} E_2$$

Therefore

$$(C_1 s + G_2 + G_3) E_1 - G_3 E_2 = 0$$

$$-G_3 E_1 + \left(G_3 + \frac{1}{L_4 s}\right) E_2 = 0$$

Impedance and Admittance

In the example above, Cs and $1/Ls$ played the role of conductance. To distinguish these from a true conductance, we call them "admittances." Likewise, $1/Cs$ and Ls are "impedances," which are like resistances:

Element	Impedance	Admittance
Resistor	R	$G = 1/R$
Capacitor	$1/Cs$	Cs
Inductor	Ls	$1/Ls$
General	Z	$Y = 1/Z$

Can write down the equations for any RLC circuit <u>by inspection</u>, just by treating L's and C's like R's!