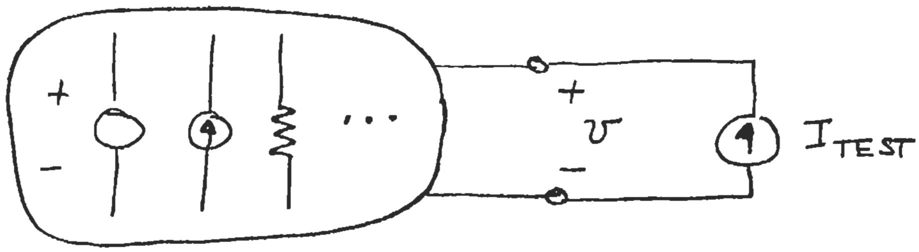
Thevinin and Norton Equivalent Networks



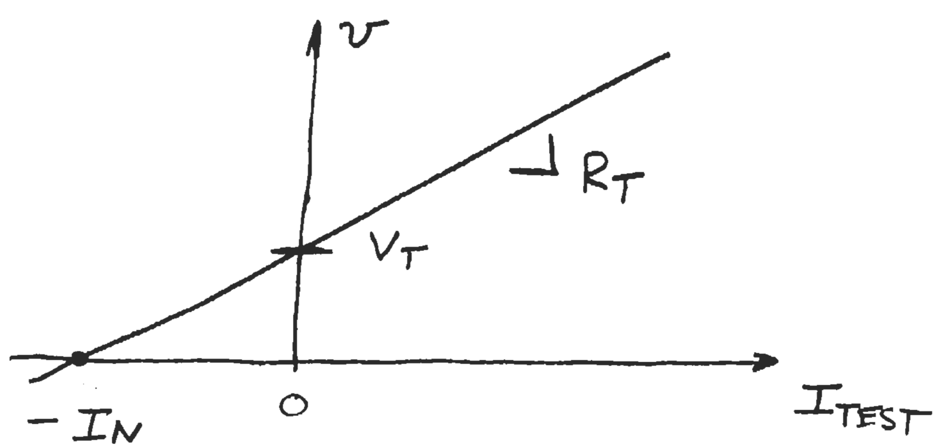
If we control current at terminal, what is voltage?

Analyze by superposition!

$$v = a_1 V_1 + a_2 V_2 + \cdots + b_1 I_1 + b_2 I_2 + \cdots + R_T I_{T1}$$

- a_1, b_1, R_T: constant
- V_1: internal voltage source
- I_1: internal current source
- I_{T1}: external test current

So $v = V_T + R_T I_{TEST}$



V_T = Thevinin equivalent voltage
= "open circuit voltage"

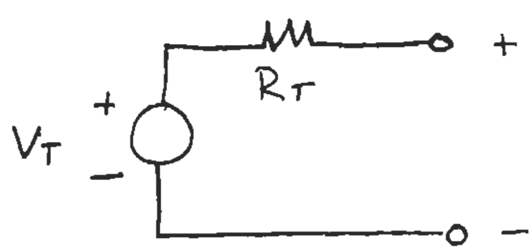
R_T = Thevinin equivalent resistance
= "output resistance" (or "impedance")

I_N = "short circuit current"

Any *two* of the parameters above completely characterize the behavior of the circuit *at the terminals*

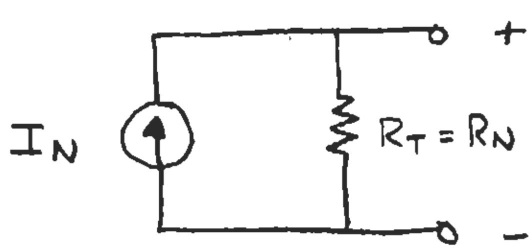
$R_T = V_T / I_N$

Circuit with open-circuit voltage V_T, output resistance R_T:



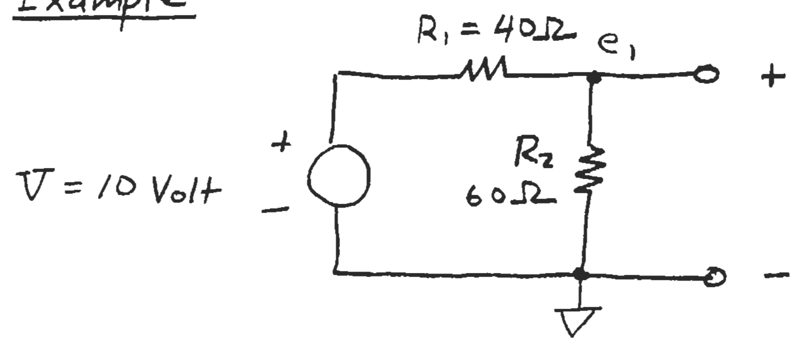
Thevinin equivalent network

Circuit with short-circuit current current I_N, output resistance R_T:



Norton equivalent network

Example



What is Thevinin equivalent?

Method 1:

Find V_T = open circuit voltage

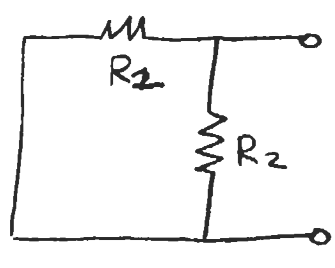
$$= \frac{R_2}{R_1 + R_2} \cdot V$$

$$= \frac{60}{40 + 60} \cdot 10 \text{ volt} = 6 \text{ volt}$$

(could use node method)

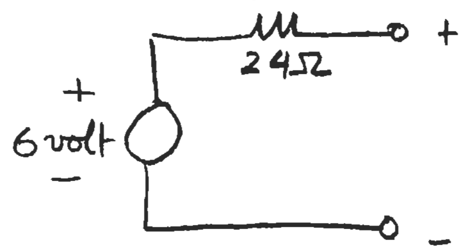
Find R_T = output resistance with all sources = 0

$R_T =$

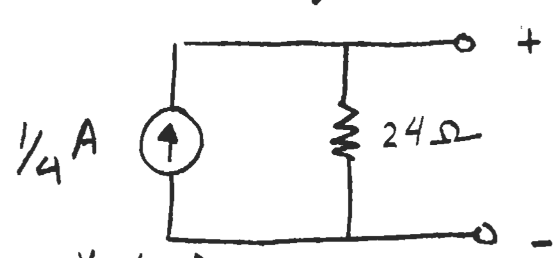


$$= R_1 \| R_2 = \frac{40 \cdot 60}{40 + 60} = 24\Omega$$

Thevinin equivalent:

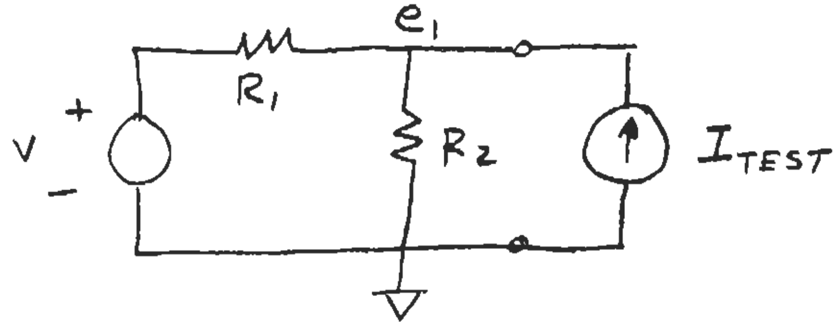


Norton Equivalent:



$(I_N = V_T / R_T)$

Method 2: Node method



$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) e_1 - \frac{1}{R_1} V - I_{TEST} = 0$$

$$\Rightarrow \qquad e_1 = \left(\frac{1}{R_1} V + I_{TEST}\right) \Big/ \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \frac{R_2 V + R_1 R_2 I_{TEST}}{R_1 + R_2}$$

$$\Rightarrow V_T = \frac{R_2}{R_1 + R_2} V = 6 \text{ volt}$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = R_1 \| R_2 = 24\,\Omega \qquad \checkmark$$