Muddy Card Responses Lecture M14

Why are we rotating coordinate systems? There are many reasons. Here are two. Materials tend to fail as a result of the stresses acting on them. Brittle materials fail due to the maximum tensile stress, ductile materials (including most metals) fail due to the maximum shear stress. As part of assessing structural integrity it would be useful to be able to determine the maximum values of stress at a particular location. A second reason is that we sometimes need to specify directions for making joints between sheets of material, or for putting in structural reinforcement (stiffeners or fibers in a composite) and we will want to be able to analyze the stresses as a function of direction.

Unclear as to why we need to transform the stresses onto a different set of axes. See comments above.

Other than maximum and minimum stresses what information does Mohr's circle give us/ what is the point of drawing it? How do you get equations from it? The maximum and minimum stresses are pretty important in their own right. Being able to resolve stresses into different directions is also important. If you draw a Mohr's circle in terms of an arbitrary state of stress and then look at the state of stress at an angle θ from the original axis directions, you will recover the equations we derived from the triangular element.

Would we ever transform to a different coordinate system, ie. From rectangular to polar? Or are transformations simply for orthogonal coordinates. If so why do we transform in the first place? You can certainly transform coordinates from cartesian to polar, but we are not going to examine that in Unified. The reasons for needing to being able to rotate coordinates are given above.

Mohr's circle – needs more exposure and explanation of what it does. Understood. We will cover it in recitation tomorrow and also do some examples in lecture.

Still a little fuzzy on how to get stresses from reading Mohr's circle. See above. We will do more examples.

Need to think more about Mohr's circle – still a little confused about the signs. See above. We will do more examples.

How do you read the results off a Mohr's circle diagram? See above.

Confused about Mohr's circe, Assume we will go over more Mohr tomorrow. Indeed we will.

How to use Mohr's circle once you've got it drawn. We will examine tomorrow.

Tensor form transformation?? We will cover this in recitation.

How is tensor form any different from equilibrium? It seems they would derive the same equation using same process. I am not entirely sure what the question is here, but whether you rotate stress by considering the equilibrium of a triangular element, use direction cosines

or Mohr's circle, you will end up with identical results - otherwise there would be something wrong. Direction cosines lend themselves most readily to writing a computer code to do the transformation for you. Mohr's circle is the easiest way to visualize the stress state.

More info on direction cosines and its use with tensor form See above.

When working with the stresses and shear that at act at angles to the faces, is it necessary to alter the axis to the $\tilde{x}$ plane, or can we instead componentalize the *shẽar* and *strẽss* to the x_1, x_2 and x_3 axis? Not sure if I quite understand this question. I think that you are asking whether you can rotate the axes in either direction, ie from $\tilde{x}_n$ to x_m or back again. Of course this is correct. The definition of "rotated" and "unrotated" axes is entirely arbitrary.

For the transformation you use dx_2 and σ come from? Not sure what the question is here. I think that you are asking more about how we set up the equilibrium of the triangular element. The idea is to set up the equilibrium of an element that reveals how the stresses vary with direction.

Can you draw the shear stresses in any arbitrary direction?
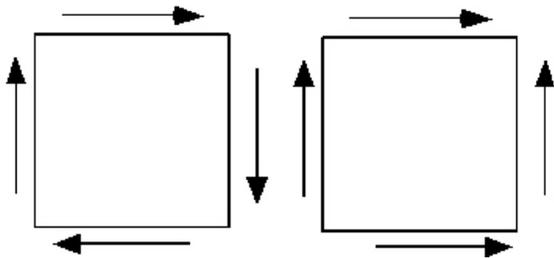
No we cannot draw the shear stresses in arbitrary directions. Neither of these stress states are in equilibrium. The one of the left is clearly going to rotate. And the one on the right is neither in equilibrium in the x_1 nor the x_2 directions.

Why did we define stresses moments to be positive in the clockwise direction? Previously you defined moments in the counter clockwise direction to be positive. This convention is just for the purpose of drawing Mohr's circle, so that it conforms to the formulae for rotating the stresses we derived by considering the equilibrium of the triangular element.

What is the exact reason why we pair σ_{12} and σ_{11} together and σ_{21} and σ_{22}? There are two reasons. 1) This is what it takes to make the Mohr's circle representation of the equations we derived by considering the equilibrium of the triangular element work. 2) σ_{12} and σ_{11} are the stresses acting on the x_1 face, which defines a direction on the Mohr's circle and similarly for σ_{21} and σ_{22}.

What is the exact reason why we pair σ_{12} and σ_{11} together and σ_{21} and σ_{22}? Why are they plotted on the same axis? See above. They are plotted on the same axis (I assume that

you mean the same diameter of the circle) because in Mohr's circle space the angles are doubled, so in the physical world x_1 and x_2 are 90° apart, on Mohr's circle they are 180° apart.

$\tilde{\sigma}_{mn} = \ell_{\tilde{m}p}\ell_{\tilde{n}q}\sigma_{pq}$ **- what is this.?** This refers back to lecture U2/3 . $\ell_{\tilde{m}p}$ etc are the direction cosines, which we now see are really a second order tensor. It takes two direction cosines to transform (rotate) a second order tensor quantity such as stress. Mohr's circle provides a simple graphical means of doing this in 2-D.

For tensor form, in $\tilde{\sigma}_{mn} = \ell_{\tilde{m}p}\ell_{\tilde{n}q}\sigma_{pq}$, what do p and q correspond to? p and q correspond to the x_p (or x_q) original (unrotated)axis system, as opposed to the $\tilde{x}_q$ rotated axis system. p, q, $\tilde{m}$, $\tilde{n}$ take on values of 1, 2 and 3 – since this is a 3-D problem.

What do the p and q subscripts mean? Where do they come from? See above. The key point to remember is that the letters used as indices, whether they are m and n or p and q they take on values of 1, 2 and 3 (for a 3-D problem).

Is there a Mohr's sphere? No. If you want to transform 3 -D stress you need to know one of the principal directions before you start and then you can use Mohr's circle in 2-D we may discuss this at the end of this module if there is time, or if someone asks me.

So for the most part can you treat most of the stress problems as trusses, and then after solving for the unknown forces you can solve for stresses? I would not exactly phrase it like this. However, the process is that you convert the stresses to forces, and then apply equilibrium of forces

How does the transformation of stress yield that $\cos^2$ term? I hope that by doing problem M12 you will understand this for yourself. Essentially one angle comes from projecting the areas onto the new axis system, and the second angle comes from projecting the force.

You made references to the two angles in the equilibrium equations but aren't they the same angle? Yes they essentially are the same angle, but it appears twice in each expression, either in sine or cosine form.

What are the new and old axes by your definition? m or $\tilde{n}$ are new? It could be either. But to transform a first order tensor from the x_m to the $x_{\tilde{n}}$ we introduce the direction cosine tensor, $\ell_{\tilde{n}m}$ which is defined such that: $\ell_{\tilde{n}m} = \cos(\overline{x_{\tilde{n}}x_m})$ i.e. the cosine of the angle between axis $x_{\tilde{n}}$ and axis x_m . If we then want to transform a first order tensor (i.e. a vector) such as a force this is given by:

$$F_{\tilde{n}} = \ell_{\tilde{n}m}F_m$$

where if this were to be expanded we would have:

$$F_{\tilde{1}} = \ell_{\tilde{1}1}F_1 + \ell_{\tilde{1}2}F_2 + \ell_{\tilde{1}3}F_3$$

$$F_{\tilde{2}} = \ell_{\tilde{2}1}F_1 + \ell_{\tilde{2}2}F_2 + \ell_{\tilde{2}3}F_3$$

$$F_{\tilde{3}} = \ell_{\tilde{3}1}F_1 + \ell_{\tilde{3}2}F_2 + \ell_{\tilde{3}3}F_3$$

What are direction cosines? See above.

Generally, no mud... All the subscript action can get confusing... Understood. After a while and a little exposure it should start to feel more logical.

Amount of confusion is directly proportional to the number of different little symbols you have on the board. Overwhelming See comments above.

What is a first order tensor?: That is a vector quantity - only one index is required to specify the direction of a vector.

Way to fast at the end of lecture (last 15 minutes). I agree with this. I wanted to get to the basics of Mohr's circle, because I think that it helps if you have a chance to think about concepts such as this overnight. We will spend more time on it tomorrow.

9 more days till Thanksgiving!!! Too right!