Muddy Card Responses Lecture M5 2/11/2004

Synopsis. Introduced assumption that plane sections remaining plane. Used this assumption to provide an expression for the variation in axial displacement, u, as a function of z position through the thickness of a beam. Differentiated u(x,z) to obtain axial strain, exx, then applied elastic constants to obtain stresses. Identified discrepancy due to absence of shear strains (from plane sections assumption). We will examine this later. Finished by showing that the integral of the axial stress and shear stress over the cross-sectional area will give us the resultant axial force and shear force on the cross-section.

A beam bends, but how can there be no shear force on the top surface if it is stretching? Remember stretching implies an axial strain (not a shear strain). A shear strain would imply a change in angle, but no stretching. Thus your intuitioon that the top surface stretches is consistent with the idea that there are no shear stresses, only axial stresses.

Curious how does $\varepsilon = -z\dfrac{d^2w}{dx^2}$ translate to the curvature of the beam? The quantity $\dfrac{d^2w}{dx^2}$ is the curvature of the beam. Curvature is the second derivative of displacement (position).

Are τ_{xz}, and σ_{xx} the only stresses in the beam? Yes – we will see this tomorrow.

Sign convention on shear just got confusing. Oh dear. Please don't let it interfere with your understanding of what is physically happening – it is just a convention that allows us to be consistent with the directions the stresses, forces, moments, strains and displacements act.

Feb 11 (Loretta's Birthday) Forgive me, if I had known we could have sung for her! **I think that it just needs time to sink in! Maybe some example problems for applying concepts? It's one thing to see the variables another to really know how to use them. Too much ridiculous geometery!! Ahhh!** Relax. You can do this. The geometry is only to do with the displacements on a cross-section undergoing a rigid body rotation.

How good are these approximations? I.e. how can plane sections really remain plane? If I was building a skyscraper don't I need real bending of the beam not just approximations? Simple beam theory really works very well and is a very powerful analytical tool. You will do a lab later in the semester to examine this for yourselves. Simple beam theory is used extensively in civil engineering.

You lost me on that lass PRS question. Can you explain why $S = \displaystyle\int_{-h/2}^{h/2} \sigma_{xz}b\,dz$ again?

Actually the correct answer was $S = -\displaystyle\int_{-h/2}^{h/2} \sigma_{xz}b\,dz$ due to the sign convention. Basically we are integrating up the effect of the shear stress acting on differential elements of area bdz to obtain the equivalent total force. It would be helpful if you could let me know what your best guess was regarding what the answer should have been.

Could you explain again the shear strain/stress inconsistency ? I don't understand why there is an inconsistency. We will cover this over the next couple of lectures.

Is the proposed circular curvature and actuality or merely an ideal? It seems it would ideally behave this way and could be modeled as such but in practice curves are a more elliptical function. We will examine this over the next couple of lectures.

Still not sure about 1ˢᵗ PRS (actually the second) why isn't it sinϕ? It should be tanϕ – think about what dw/dx represents or the triangle formed by the vector u and the vertical distance z.

So there is a deformation in the –x direction when moments are exerted at each end of the beam. Yes. Think about the related problem of the deformations of a cantilever truss.

Look into M5 at the end was a little unclear but that is expected. Fine, we will revisit this tomorrow.

I got lost towards the end. Wit the integration of shear stress. See above.

What is the significance of all this? I mean how is it used? Is it for beams? Are there any practice problems you could refer us to?

Because $\dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \sigma_{yz}}{\partial y} + \dfrac{\partial \sigma_{zx}}{\partial z} = 0$ **don't mean anything to me and I feel lost. What are the really important concepts underlying these equations? I sense frustration here. Please come and talk to me.** This is all about beams, and beams are a good idealization of many structures which we are interested in, notably wings (but also blades in helicopters or gas turbines). The underlying principles are still equilibrium, compatibility and constitutive behavior.

How are there no shear stresses? There is tension on top of the beam and compression on the bottom? Wouldn't that mean that the angles are changing? (a picture was drawn). No. Think about an initially straight beam deforming into a circular arc. The outer surfaces of the beam go into tension or compression. The cross-sections initially perpendicular to the center line of the beam become radial planes, remaining perpendicular to the center line of the beam. Thus the angle does not change, it remains as 90°.

When you bend a beam the top is in tension, the bottom is in compression, won't this cause displacements? Top gets longer, bottom gets shorter? Will this cause strain? Yes absolutely. We will examine this fully tomorrow.

I would like to treat more applications, how to find min pt of beam after deformed. We will get on to this.

A chain hanging from two pins at equal height takes what general shape? What is the name for that shape? It is called a "catenary" and is a cosh function. See http://www-gap.dcs.st-and.ac.uk/~history/Curves/Catenary.html for a historical reference. http://www.du.edu/~jcalvert/math/catenary.htm provides a derivation.

You wrote $u = -z\dfrac{dw}{dx^2}$ **on the board. Shouldn't it be dx and not dx^2.** Yes, absolutely, my error.

I didn't fully understand the wording of the second PRS question. Could you explain in easier terms what you were asking for? The PRS question is posted with these muddy responses. Take a little more time to evaluate the question and if it does not make sense please ask me.

I loose my attention when most of the lecture is on the overhead. Thanks for the feedback. I am not entirely comfortable with lecturing off the overhead myself. I am trying to find the balance between using the board for the most important concepts, the PRS questions to reinforce them and the overhead+printed notes for "routine" manipulations.

Most important: Euler-Bernoulli Deformation assumption? Yes. **The implication of it is that it makes calculation of stress and strain in beams simple?.** In beams, yes.

Why is there shear force but not shear strain? This is an inconsistency that arises from the Euler-Bernoulli assumption. We will reexamine this shortly.

Can you explain how you got $u = -z\dfrac{dw}{dx}$**? Why multiply** $\dfrac{dw}{dx}$ **by z to get u? isn't u just** **–zsinϕ?** Please refer to the notes. The key figure is this one:
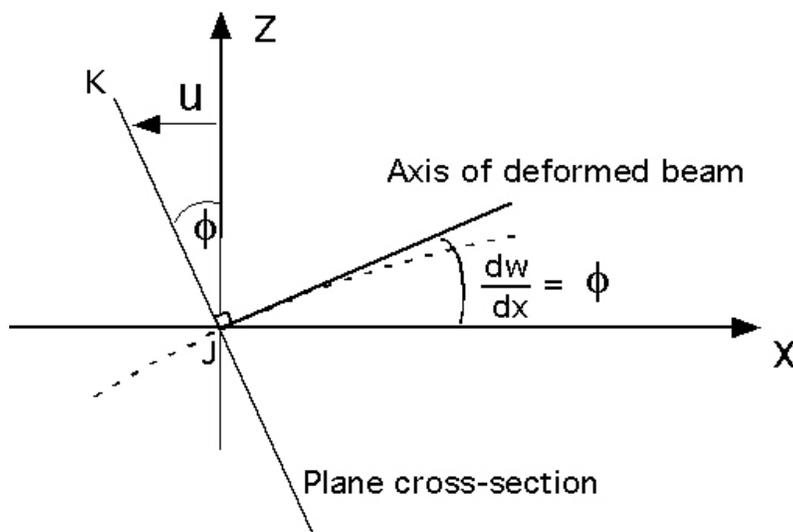
Perhaps a clearer picture of what we are talking about would have been:
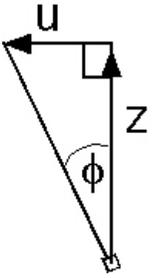
Here I hope that it is obvious why: $u = -z \tan \phi$ and given $\dfrac{dw}{dx} = \tan \phi \approx \phi$ we obtain

$u = -z\dfrac{dw}{dx}$. We want the expression in this form so we can relate w(x) to u(x) and thence to stress and strains in the beam.

Is there a step by step way to work completely through a beam problem? There are so many equations. I am not sure what to do with each of them. I guess even a quick explanation of what each equation is for would be helpful. Relax. We will do this. However, you can help yourself by taking the time to understand the steps in the derivation of beam theory – it is important to understand the modeling assumptions and the steps in the model.

Do we ever use curvature κ or do we only refer to it in terms of $z\dfrac{d^2w}{dx^2}$? We may use κ,

please note that the curvature is actually: $\dfrac{d^2w}{dx^2}$ (omit the "z").

Most important: planar relationships of beams:. Muddiest, all the little subscripts mess with my dyslexia. My apologies, it is tough to do structural mechanics with out subscripts – too many tensor quantities!

Confused about the displacement due to moments. What is w(x)? w(x) is the displacement in the z direction expressed as a function of x.

So if there is no strain then there is no stress. I thought that you could have strain without stress. You are correct, in general you can have strain without stress and vice versa. However , I was talking about shear stresses and strains in an isotropic material in which case $\tau = G\gamma$ and shear stresses imply shear strains and vice versa.

Confused about the details of how we got $\dfrac{dw}{dx}$ but I'll read the book and it'll probably be fine. I like your attitude – I appreciate it.

How is $\dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \sigma_{zx}}{\partial z} = 0$ **derived.** From last term we had the differential equation of

equilibrium: $\dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \sigma_{yz}}{\partial y} + \dfrac{\partial \sigma_{zx}}{\partial z} = 0$. Since $\sigma_{yz} = 0$ we obtain: $\dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \sigma_{zx}}{\partial z} = 0$.

Just for the record, when applying pure moment in what directions are they to the beam. Stresses and in what directions there strains? (this is the question as written – I am interpreting). For a pure moment there is only σ_{xx}. Obviously for there to be a shear force there must be a shear stress. Note, for the case of a pure moment, there is no shear force and therefore no shear stress.

Don't understand the negative sign of the last PRS: $S = - \displaystyle\int_{-h/2}^{h/2} \sigma_{xz} b\, dz$. This is just because our sign convention for a positive shear stress is opposite to our sign convention for a positive shear force.

There were 4 muddy cards with no mud, or positive responses.