Muddy Card Responses Lecture M12

Just what does shear stress do? It pulls/pushes on a face making it want to crack apart? Not really. It is the stress that is generated if you were to put your hand on the face of the cube and try to slide your hand across the face, the friction between your hand and the surface would result in a shear stress.

Note there are some lecture notes missing from the online postings, will these be available (preferably before Friday) I am very sorry about this. I had prepared the notes but somehow omitted them to send them to the graduate TA's to post. They are up on the web site now.

Could you expand on how you sum stresses. Not sure what you mean here. I can interpret it in two ways. 1) the stresses at a particular point simply sum, in the same way as any other linear variables. If you apply a stress σ_{mn} and then an additional stress σ_{mn} then the total stress, at that point, is $2\sigma_{mn.}$ 2) The other interpretation is that if you are summing stresses, in the sense of ensuring equilibrium, then you must remember that you need to convert the stresses to forces, by multiplying by the area that they act on, in order to apply equilibrium.

When we do equilibrium of forces why does only one of each pair of strains vary?

$$\left(\sigma_{11} + \frac{\partial \sigma_{11}}{\partial x_1}\delta x_1\right)(\delta x_2 \delta x_3) - \sigma_{11}(\delta x_2 \delta x_3) +$$

First, we are dealing with stresses not strains. These are two very different quantities and it is important to be precise.

Second, we (I) have set this problem up so that there is an arbitrary gradient of stress across the element. So I have chosen there to be a stress of σ_{11} on the negative x_1 face and $\left(\sigma_{11} + \frac{\partial \sigma_{11}}{\partial x_1}\delta x_1\right)$ on the positive x_1 face. I have also chosen where I place the inifinitessimal cuboid relative to the coordinate systems. This is all done to make the analysis easy/convenient. Note that the choice of coordinate system, etc. does not alter the physics of the problem (i.e. that equilibrium must be maintained).

Still a little confused on how stresses are varying. How should we think of it physically? The equation $\frac{\partial \sigma_{mn}}{\partial x_m} + f_n + 0$ provides a relationship between how individual components of stress vary with position over a structure. Basically it says that if one component of stress varies with position, the other components of stress must also vary for that element of material to remain in equilibrium. For instance, in a 2-D system, if σ_{11} varies with x_1 then this implies that σ_{21} must also vary with x_2.

Second PRS questions not clear. Take a look at it on line together with the notes, and see if it makes more sense. If not feel free to ask.

Why is the σ_{mn} the on the other (opposite) faces of the cube negative? Isn't it the normal direction also pointing out so shouldn't $\sigma \cdot \hat{n}$ be positive? This is an important question. This returns to the issue we met when we were talking about bar forces. Positive stresses are tensile, inasmuch as they tend to act in positive directions on positive faces of our infinitesimal cuboid. However, when we come to apply equilibrium, we have to consider their absolute direction, so a force due to a stress acting in a positive x_1 direction is positive in the equilibrium equation, and that acting in a negative x_1 direction is negative.

How do you represent different stresses on opposite faces?

How do you determine the number of independent equations and number of terms from the tensor equation in the PRS question? See below:

$\sigma_{mn} = E_{mnpr}\varepsilon_{pr}$ when fully expanded becomes:

$$\sigma_{11} = E_{1111}\varepsilon_{11} + E_{1112}\varepsilon_{12} + E_{1113}\varepsilon_{13} + E_{1121}\varepsilon_{21} + E_{1122}\varepsilon_{22} + E_{1123}\varepsilon_{23} + E_{1131}\varepsilon_{31} + E_{1132}\varepsilon_{32} + E_{1133}\varepsilon_{33}$$

$$\sigma_{12} = E_{1211}\varepsilon_{11} + E_{1212}\varepsilon_{12} + E_{1213}\varepsilon_{13} + E_{1221}\varepsilon_{21} + E_{1222}\varepsilon_{22} + E_{1223}\varepsilon_{23} + E_{1231}\varepsilon_{31} + E_{1232}\varepsilon_{32} + E_{1233}\varepsilon_{33}$$

$$\sigma_{13} = E_{1311}\varepsilon_{11} + E_{1312}\varepsilon_{12} + E_{1313}\varepsilon_{13} + E_{1321}\varepsilon_{21} + E_{1322}\varepsilon_{22} + E_{1323}\varepsilon_{23} + E_{1331}\varepsilon_{31} + E_{1332}\varepsilon_{32} + E_{1333}\varepsilon_{33}$$

$$\sigma_{21} = E_{2111}\varepsilon_{11} + E_{2112}\varepsilon_{12} + E_{2113}\varepsilon_{13} + E_{2121}\varepsilon_{21} + E_{2122}\varepsilon_{22} + E_{2123}\varepsilon_{23} + E_{2131}\varepsilon_{31} + E_{2132}\varepsilon_{32} + E_{2133}\varepsilon_{33}$$

$$\sigma_{22} = E_{2211}\varepsilon_{11} + E_{2212}\varepsilon_{12} + E_{2213}\varepsilon_{13} + E_{2221}\varepsilon_{21} + E_{2222}\varepsilon_{22} + E_{2223}\varepsilon_{23} + E_{2231}\varepsilon_{31} + E_{2232}\varepsilon_{32} + E_{2233}\varepsilon_{33}$$

$$\sigma_{23} = E_{2311}\varepsilon_{11} + E_{2312}\varepsilon_{12} + E_{2313}\varepsilon_{13} + E_{2321}\varepsilon_{21} + E_{2322}\varepsilon_{22} + E_{2323}\varepsilon_{23} + E_{2331}\varepsilon_{31} + E_{2332}\varepsilon_{32} + E_{2333}\varepsilon_{33}$$

$$\sigma_{31} = E_{3111}\varepsilon_{11} + E_{3112}\varepsilon_{12} + E_{3113}\varepsilon_{13} + E_{3121}\varepsilon_{21} + E_{3122}\varepsilon_{22} + E_{3123}\varepsilon_{23} + E_{3131}\varepsilon_{31} + E_{3132}\varepsilon_{32} + E_{3133}\varepsilon_{33}$$

$$\sigma_{32} = E_{3211}\varepsilon_{11} + E_{3212}\varepsilon_{12} + E_{3213}\varepsilon_{13} + E_{3221}\varepsilon_{21} + E_{3222}\varepsilon_{22} + E_{3223}\varepsilon_{23} + E_{3231}\varepsilon_{31} + E_{3232}\varepsilon_{32} + E_{3233}\varepsilon_{33}$$

$$\sigma_{33} = E_{3311}\varepsilon_{11} + E_{3312}\varepsilon_{12} + E_{3313}\varepsilon_{13} + E_{3321}\varepsilon_{21} + E_{3322}\varepsilon_{22} + E_{3323}\varepsilon_{23} + E_{3331}\varepsilon_{31} + E_{3332}\varepsilon_{32} + E_{3333}\varepsilon_{33}$$

The process is quite logical (albeit tedious to write out) Note, since we know that $\sigma_{mn}=\sigma_{nm}$ we might suspect that there will actually only be 6 independent equations -which turns out to be true. However, in the absence of that information there would in general be the 9 equations, containing 81 terms shown above. Kudos to those of you who said 6 for this reason!.

What does $\sigma_{mn} = E_{mnpr}\varepsilon_{pr}$ represent?? This is the 3-D equation of elasticity, which represents the constitutive law for elastic materials. I was just using it as an illustration of how the tensor representation provides a succinct presentation of the equation.

I am confused about what the equation form the last PRS question represents. What do the equations physically represent and how do I find them. See answer above. We will meet this equation again later in the term. Don't worry about it for the time being.

What are p and r in the last concept question? They are indices which take values of 1, 2 and 3. Just the same as if they were m and n**.**

Explain last PRS, how did you know p and r take values of 1, 2 and 3? The convention for indicial (tensor) notation is that roman letters (a throug z) take values of 1, 2 and 3 when used as subscripts – which corresponds to 3-D problems. If we are working in 2-D then we use Greek letters (α through ω)

What is $\sigma_{mn} = E_{mnpr}\varepsilon_{pr}$ see above.

Can you re-explain the last PRS question. See above.

Please explain $\sigma_{mn} = E_{mnpr}\varepsilon_{pr}$ concept question again. See above.

If m and n can generate 9 permutations and p and r the same, then why don't we get 81 permutations in total? The difference is that p and r are repeated indices, so the 9 permutatons result in 9 terms that are added together in each equation. m and n are not repeated (in the term $E_{mnpr}\varepsilon_{pr}$), which means that each permutation of m and n generates a separate equation.

Tensor expression at the end. I'll look on line. I hope that this helps.

Is there any way that you can post the notes for each lecture prior to that lecture? I am very sorry about this. My apologies for this, I had written them and just forgot to send them to the TA's.

How did you determine the gradients $\frac{\partial \sigma_{mn}}{\partial x_m} dx_m$? These are arbitrary. We will make use of this next term as we start to look at structural elements in which there can be stress gradients. For the time being we are just setting up the mechanics so that we allow for the fact that there can be stress gradients at all.

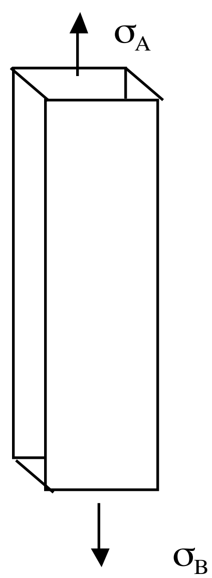
What do you mean by "extension $\sigma_{12}=\sigma_{21}$ etc". I meant that we could apply equilibrium of moments about the x_2 and x_3 axes and we would have found that the other two pairs of shear stresses must be equal (i.e. $\sigma_{12}=\sigma_{21}$ and $\sigma_{13}=\sigma_{31}$).

What does $\sigma_{mn}= \sigma_{nm}$ really mean in physical terms". It means that the shear stresses acting on orthogonal faces to cause rotation about the same axis must be equal in order for the differential element that they are acting on to be in moment equilibrium. These pairs of shear stresses are sometimes called "complementary" shear stresses.

I am not quite clear as to how we would use this numerically. I guess we will have examples in the homework. Yes.

In the first PRS question should the answer have been $\sum_m \frac{\partial \sigma_{mn}}{\partial x_m} + f_n$ instead of $\frac{\partial \sigma_{mn}}{\partial x_m} + f_n + 0$? Yes,this is correct, but you have to remember that there will be three separate equations, corresponding to n=1, 2, and3.

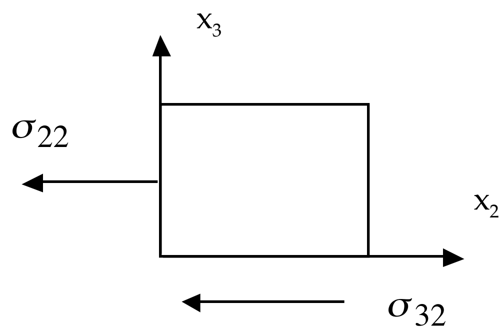
How do you represent different stresses on opposite faces?



Do you average and call that σ_{33} Is it OK to do this for shear forces too?

Good question, in a sense this is the whole point of $\frac{\partial \sigma_{mn}}{\partial x_m} + f_n + 0$. In the picture you have drawn if σ_A and σ_B are different and these are the only stresses acting on the structure (the rectangular bar) then the bar cannot be in equilibrium, there must be other stresses (for instance shear stresses on the side faces) that maintain the overall equlibrium.

How are $\sigma_{22}\,\sigma_{32}$ **positive** (in figure below)?



σ_{22} acts on the negative x_2 face (i.e. the face with the negative x_2 direction as its normal) in the negative x_2 direction. This makes it a positive (i.e tensile stress) – it is pulling out of the element. This is the same convention that we used for tensile forces in bars of trusses.

Identical logic applies to σ_{32} it acts on the negative x_3 face in the negative x_2 direction, which makes it a positive shear stress.

28 cards indicating no mud, or encouraging remarks (and a couple stating a need for sleep, and a couple hoping my cold improves). Thank you