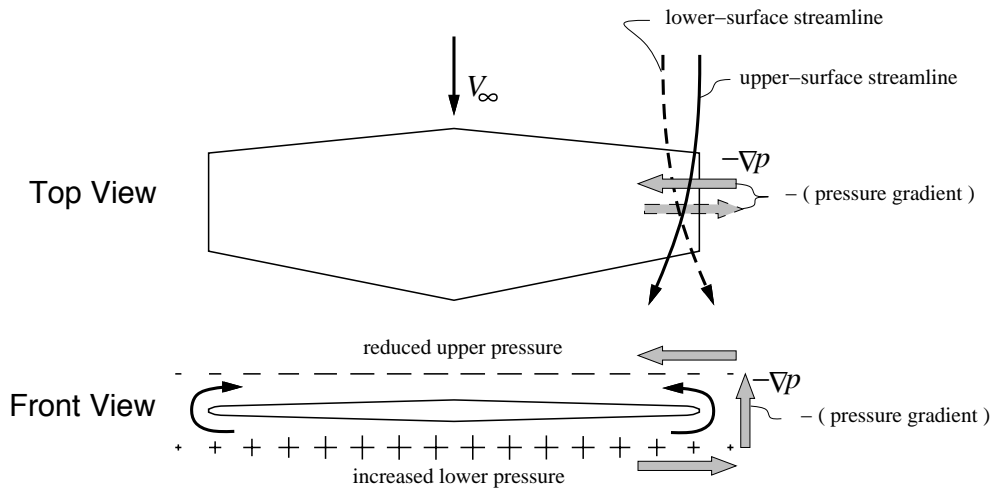# Fluids – Lecture 5 Notes

1. Introduction to 3-D Wings
2. 2-D and 3-D Coefficients

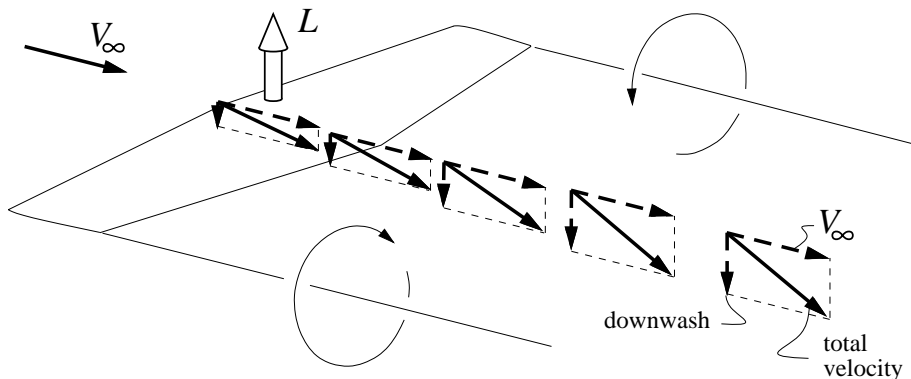Reading: Anderson 5.1

Introduction to 3-D Wings

A 2-D airfoil can be considered as a wing of infinite span, with each spanwise location identical. In contrast, on a 3-D finite-span wing, the presence of wingtips introduces spanwise flow variations, a number of important new physical effects.

Spanwise flow. An upward-lifting wing must on average have a pressure difference between its upper and lower surfaces. Outside of the wingtips, this pressure difference must disappear. Consequently, fluid particles which approach the wintip above the wing are subjected to a spanwise pressure gradient which causes them to curve towards the wing center. Fluid particles which approach below the wing will curve away from the wing center. Particles close to the tip will tend to flow around the tip edge, from the lower to the upper surface.

Trailing vortices. The flow curling around the tip forms a *tip vortex*, which then trails downstream, and can be easily made visible with smoke visualization.

Downwash. The velocity field associated with the two trailing vortices is mostly downward (opposite to lift direction) directly behind the wing, and upward outside the tip vortices. The downwash can also be viewed as the result of the lifting wing "pushing down" on the air, which results in the air having added downward momentum directly behind the wing.

<u>Induced angle and lift reduction.</u> There is a nonzero downwash velocity $-w$ at the wing itself (w is positive up). In other words, the wing operates in its own downwash. The apparent freestream velocity which the wing sees is therefore tilted by the *induced angle*

$$\alpha_i \;=\; \arctan\frac{-w}{V_\infty} \;\simeq\; \frac{-w}{V_\infty}$$

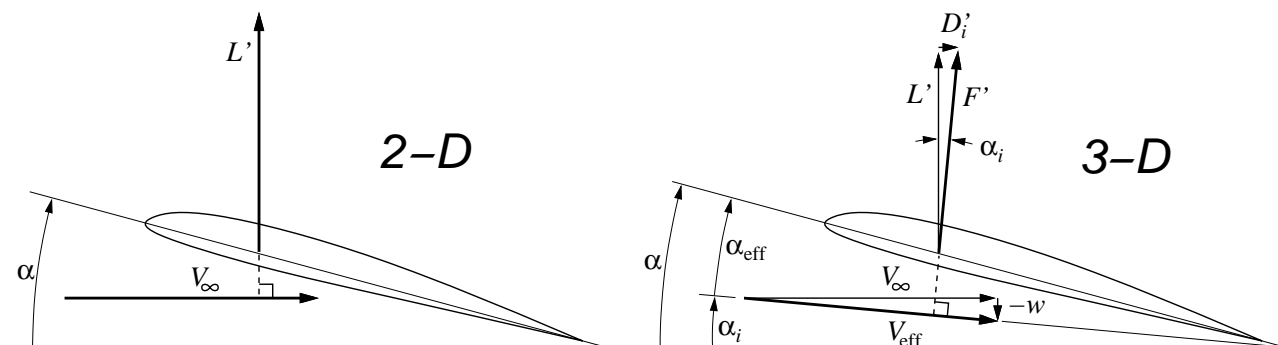where the small-angle approximation is justified since

$$|w| \;\ll\; V_\infty$$

for typical wings. The effective angle of attack that the 3-D wing sees is significantly reduced from the geometric angle of attack α,

$$\alpha_{\text{eff}} \;=\; \alpha \;-\; \alpha_i$$

while the net effective freestream speed V_{eff} is nearly the same as the freestream speed.

$$V_{\text{eff}} \;=\; \sqrt{V_\infty^2 + w^2} \;\simeq\; V_\infty$$

Because $\alpha_{\text{eff}} < \alpha$, the lift generated be the 3-D wing is less than if the downwash and induced angle were absent, as in the 2-D case. In practice, this means that the 3-D wing has to operate at a greater geometric α to achieve the same lift/span as the 2-D wing.

<u>Induced drag.</u> As the figure above shows, the tilt caused by the downwash velocity also tilts the lift force by the same angle α_i. The lifting force/span produced by the wing is

$$F' \;=\; \rho V_\infty \Gamma$$

which can be resolved into lift and drag components aligned with to the true freestream velocity vector $\vec{V}_\infty$. Making our usual small-angle approximations, we have
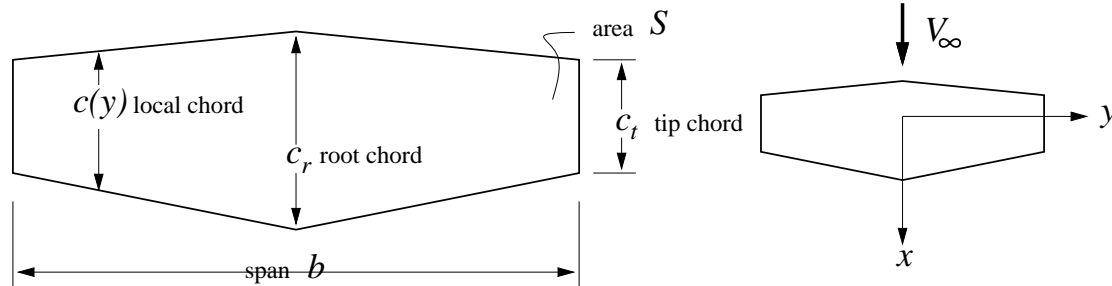
$$L' \;=\; F' \cos\alpha_i \;\simeq\; F' \;=\; \rho V_\infty \Gamma$$
$$D_i' \;=\; F' \sin\alpha_i \;\simeq\; F'\alpha_i \;=\; -\rho w\, \Gamma$$

A very important consequence of the tilting of the lift force is the appearance of *induced drag* D_i', which forms a very large fraction to the total drag on most aircraft. This induced drag is a form of *pressure drag*, and is completely unrelated to the viscous shear forces on the surface. Therefore, a lifting 3-D wing has nonzero induced drag even in inviscid flow, and is not subject to d'Alembert's Paradox.

2-D and 3-D Coefficients

Geometric parameters

The figure shows standard wing dimension terminology, and the standard coordinate system.

Several other relevant geometric parameters can be defined. One is the *average chord* c_{avg}, which is a simple average of the local chord $c(y)$.

$$c_{\mathrm{avg}} = \frac{1}{b} \int_{-b/2}^{b/2} c(y)\, dy = \frac{S}{b}$$

Another one is the *mean aerodynamic chord* $\bar{c}$, which is the root-mean-square average.

$$\bar{c}^2 = \frac{1}{b} \int_{-b/2}^{b/2} [c(y)]^2\, dy$$

For a rectangular wing, c_{avg} and $\bar{c}$ are the same, and usually differ only slightly for most tapered wings. Either choice is appropriate if consistently followed. Another important geometric parameter is the *aspect ratio*, defined as follows.

$$A\!R = \frac{b^2}{S} = \frac{b}{c_{\mathrm{avg}}}$$

Overall forces and moments

The overall forces and moments can be defined simply by integrating the appropriate unit-span quantities.

$$L = \int_{-b/2}^{b/2} L'(y)\, dy = \int_{-b/2}^{b/2} \rho\, V_\infty \Gamma(y)\, dy$$

$$D_i = \int_{-b/2}^{b/2} D_i'(y)\, dy = \int_{-b/2}^{b/2} \rho\, w(y)\, \Gamma(y)\, dy$$

$$D_p = \int_{-b/2}^{b/2} D_p'(y)\, dy$$

$$M_{c/4} = \int_{-b/2}^{b/2} M_{c/4}'(y)\, dy$$

Here, $D_p'(y)$ is the local profile drag/span.

Overall force and moment coefficients

Defining overall force and moment coefficients first requires selection of a suitable reference area and chord. Commonly-used choices are

$$S_{\mathrm{ref}} = S \quad , \qquad\qquad c_{\mathrm{ref}} = c_{\mathrm{avg}} \qquad \text{or} \qquad c_{\mathrm{ref}} = \bar{c}$$

The overall coefficients are then defined as follows.

$$C_L = \frac{L}{\frac{1}{2}\rho V_\infty^2 S_{\text{ref}}}$$

$$C_{Di} = \frac{D_i}{\frac{1}{2}\rho V_\infty^2 S_{\text{ref}}}$$

$$C_{Dp} = \frac{D_p}{\frac{1}{2}\rho V_\infty^2 S_{\text{ref}}}$$

$$C_{M,c/4} = \frac{M_{c/4}}{\frac{1}{2}\rho V_\infty^2 S_{\text{ref}} c_{\text{ref}}}$$

It is also useful to relate the overall and local lift, profile drag, and moment coefficients. Using the relations

$$L'(y) = \frac{1}{2}\rho V_\infty^2 c(y) c_\ell(y)$$

$$D'_p(y) = \frac{1}{2}\rho V_\infty^2 c(y) c_{d_p}(y)$$

$$M'_{c,4}(y) = \frac{1}{2}\rho V_\infty^2 [c(y)]^2 c_{m,c/4}(y)$$

we have

$$C_L = \frac{1}{S_{\text{ref}}} \int_{-b/2}^{b/2} c_\ell(y)\, c(y)\, dy$$

$$C_{D_p} = \frac{1}{S_{\text{ref}}} \int_{-b/2}^{b/2} c_{d_p}(y)\, c(y)\, dy$$

$$C_{M,c/4} = \frac{1}{S_{\text{ref}} c_{\text{ref}}} \int_{-b/2}^{b/2} c_{m,c/4}(y)\, [c(y)]^2\, dy$$

Therefore, the overall C_L and C_{D_p} can be seen to be the chord-weighted average of the local $c_\ell(y)$ and $c_{d_p}(y)$. In many situations, and especially in preliminary design, a common approximation is to assume that local lift and profile drag coefficients do not vary across the span, and therefore

$$C_L \simeq c_\ell \simeq \text{constant in } y$$
$$C_{D_p} \simeq c_{d_p} \simeq \text{constant in } y$$

Once the planform chord distribution $c(y)$ is fixed, the more exact integral definitions can be used.

Total drag

The total drag on a wing is the sum of profile and induced drags.

$$D = D_p + D_i$$

$$\text{or} \qquad C_D = C_{D_p} + C_{Di} \simeq c_{d_p}(c_\ell) + C_{Di}$$

In the last approximation, the $c_{d_p}(c_\ell)$ function is a 2-D airfoil drag polar, obtained from airfoil data or calculations. The induced drag coefficient C_{Di} will be considered in the next several lectures.