Fluids – Lecture 1 Notes

1. Formation of Lifting Flow

Reading: Anderson 4.5 – 4.6

Formation of Lifting Flow

Conservation of Circulation — Kelvin's Theorem

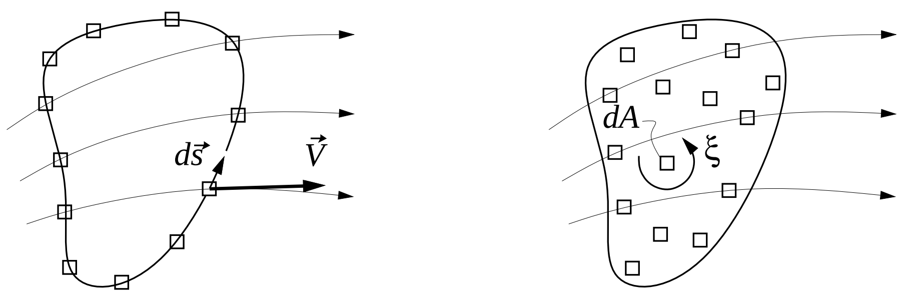
The circulation about any closed circuit is defined to be

$$\Gamma \equiv -\oint \vec{V} \cdot d\vec{s} = -\iint \vec{\xi} \cdot \hat{n} \, dA$$

where $d\vec{s}$ is an arc length element of the circuit, and $\vec{V}$ is the local flow velocity. The equivalent vorticity area integral form follows from Stokes Theorem. In 2-D, this second form is

$$\Gamma = -\iint \xi \, dA \qquad \text{(In 2-D)}$$

To investigate the formation of a lifting flow about an airfoil, we now consider the circulation Γ about a circuit demarked by fluid elements which are drifting with the flow (a fine smoke ring would constitute such a circuit). Because both the shape of the circuit and the velocities



seen by the circuit will in general change in time, there is the possibility that $\Gamma(t)$ will change in time as well. The rate of change of this circulation is

$$\frac{d\Gamma}{dt} = -\frac{d}{dt}\iint \xi \, dA = -\iint \frac{D}{Dt}(\xi \, dA)$$

where the substantial derivative has been invoked because we are seeking a time rate of change in a frame moving with the fluid. For low speed 2-D flows where density is effectively constant, the area dA of a fluid element cannot change because of conservation of mass. Hence we have

$$\frac{d\Gamma}{dt} = -\iint \frac{D\xi}{Dt} \, dA$$

But by the 2-D Helmholtz Theorem,

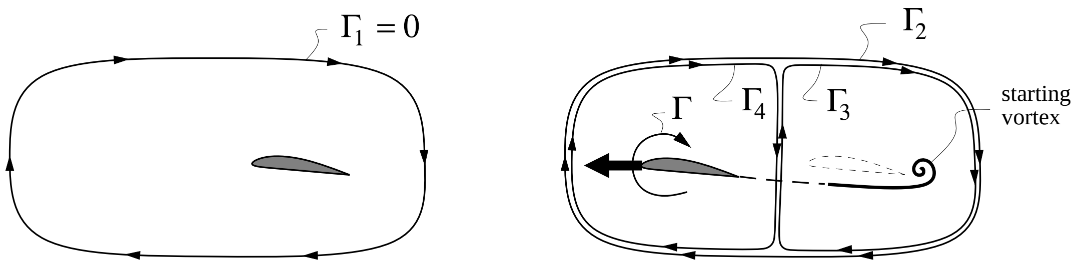
$$\frac{D\xi}{Dt} = 0$$

which then leads to *Kelvin's Circulation Theorem*

$$\frac{d\Gamma}{dt} = 0 \qquad \text{(for a drifting circuit)}$$

The Starting Vortex

Consider an airfoil initially at rest. Since $\vec{V} = 0$, the circulation Γ_1 about the circuit around



At rest **After start of motion**

the airfoil must be zero as well. When the airfoil is set into motion, it will develop a nonzero circulation, Γ_4 in the figure. Experimental flow observations show that a *starting vortex*of circulation Γ_3 is shed from the trailing edge. This shedding is associated with the Kutta condition being satisfied for every instant in time. The vortex is swept downstream as seen by an observer on the airfoil.

By construction of the circuits, we have

$$\Gamma_2 = \Gamma_3 + \Gamma_4$$

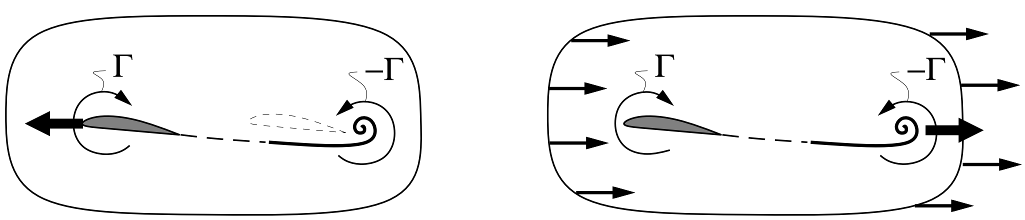
and by Kelvin's Theorem this overall Γ_2 must be the same as the outer circuit's Γ_1 before the airfoil started to move.

$$\Gamma_2 = \Gamma_1 = 0$$

Therefore, the airfoil and the starting vortex must have equal and opposite circulations.

$$\Gamma_3 = -\Gamma_4$$

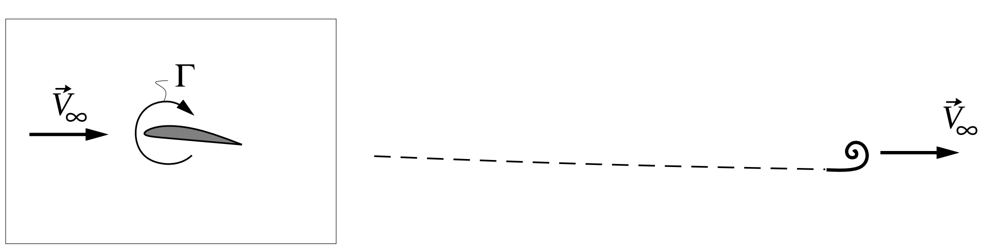
It's important to note that circulation about any circuit is the same to any non-rotating observer. Hence, Kelvin's Theorem applies to in both stationary and moving frames of reference. The airfoil and the starting vortex also have the same equal and opposite circulations in either frame.



Observer stationary **Observer moving with airfoil**

Established Steady Flow

A long time after the start of motion, the starting vortex is very far downstream behind the airfoil, and has no influence on the flowfield about the airfoil. We therefore disregard the shed starting vortex when considering any steady 2-D airfoil flow. Shed vortices must still be considered when analyzing *unsteady* airfoil flows. These are beyond scope here.



Isolated 2−D airfoil flow