Massachusetts Institute of Technology
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16.03/16.04 Unified Engineering III, IV
Spring 2004

Problem Set 11

Name: _____

Due Date: 4/27/04

	Time Spent (min)
CP13-14	
P5	
P6	
P7	
S13	
Study Time	

Announcements: Q5S will be on Friday, April 23.

C13-14

The problems in this problem set cover lectures **C13** and **C14**

1.
a.   Define a **robust** algorithm to carry out integer division using repeated subtraction. Your algorithm accepts two integers and returns the quotient and the remainder. **Hint**: What are the preconditions and postconditions of your algorithm?

b.   **Implement** your algorithm as an Ada95 program, using exception handling to provide robustness.

Turn in a hard copy of your algorithm and code listing, and an electronic copy of your code.

2.
a.   What is the **cyclomatic complexity** of the code fragment shown below?

```
loop
    exit when Flag := True;

    if A < 100 and B > 200 then
        if A > 50 then
            Sum := Sum +2;
        else
            Sum := Sum +1;
        end if;
    else
        if B < 300 then
            Sum:= Sum -1;
        else
            Sum := Sum -2;
        end if;
    end if;
end loop;
```

Hint: Draw the control flow graph

b.   What is the minimum number of test cases needed to test the fragment of code shown below? **Justify your answer**.

```
if A < 100 and B > 200 then
    if A > 50 then
        Sum := Sum +2;
    else
        Sum := Sum +1;
    end if;
else
    if B < 300 then
        Sum:= Sum -1;
    else
        Sum := Sum -2;
    end if;
```

```
        end if;
```

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Problem P5. (Propulsion) (L.O. D)

For a future high performance military fighter, with a wing loading (W/S) of 3500 N/m2, does it require does it require a higher ratio of thrust to weight to

i) Perform a steady, constant altitude, constant speed 5g combat turn at M=0.9, or

ii) Accelerate from M=0.5 to M=2 in 20 seconds?

(Note, for a 5g turn the load factor, n=5, is the acceleration perpendicular to the wing. For n=1, L=W.) Assume all the maneuvers occur at an altitude of 11km where the pressure is 22.6 kPa, the temperature is 217 K, the density is 0.34 kg/m3, and the speed of sound is 295 m/s.

Assume the coefficient of lift takes the form

$$C_D = kC_L^2 + C_{Do}$$

Where
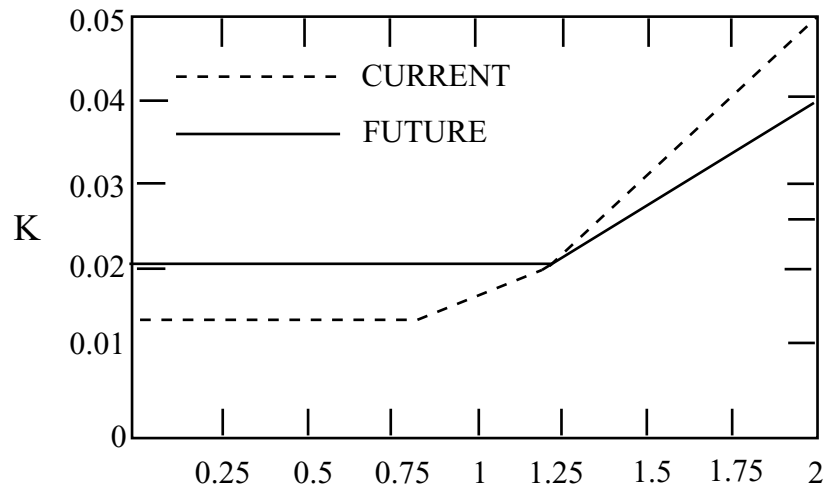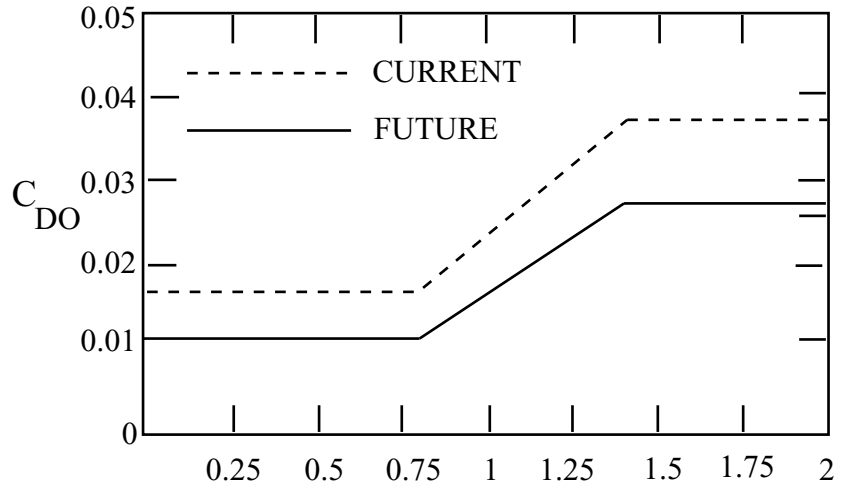
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Problem P6. (Propulsion) (L.O. E)

Evaluate the performance of a single stage sounding rocket to be used for atmospheric research.

You are given the following design parameters:

Combustion chamber temperature = 4000 K

Combustion chamber pressure = 5 MPa

Nozzle throat area = 0.0006 m2

Nozzle exit area = 0.01 m2

Propellant mass = 35 kg

Total mass at takeoff = 50 kg

Gas constant = 300 J/kg-K

Ratio of specific heats = 1.35

a) What are the Isp and take-off thrust for this rocket?

b) Assuming the rocket is launched vertically, and neglecting drag, how high will it go?

[Note: To solve this problem you will have to use relationships from thermodynamics (steady flow energy equation) and fluid mechanics (compressible flow relations). These can all be found in the Unified Propulsion notes (Chapters V and VI). Most of the relations are written in terms of the exit Mach number, Me. To find this you will have to try several values of Me in the relationship for A*/Ae until the equation gives you the same ratio as given in the problem statement.]

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Problem P7. (Propulsion) **(L.O. F)**

For this problem, consider the design of a turbojet engine for a supersonic aircraft. Assume the engine uses conventional hydrocarbon fuels (h = 42.8 MJ/kg and that $\gamma = 1.4$ and $c_p = 1000$ J/(kg-K).

There are two important material temperature limits for gas turbine engines. The first is the temperature at the inlet to the turbine (T_{T4}) which may range as high as 2000K. The second is the temperature at the exit of the compressor (T_{T3}) which is typically limited to less than 1000K. [The turbine can be run in conditions well above the melting temperature of the metal because air from the exit of the compressor is used to "cool" the turbine. This air is pumped through the turbine blades and forced out small holes in the surface forming a protective film (see picture below from Rolls-Royce, The Jet Engine). Due to their very small size, it is much harder to cool the compressor blades, so no active cooling is currently used.]
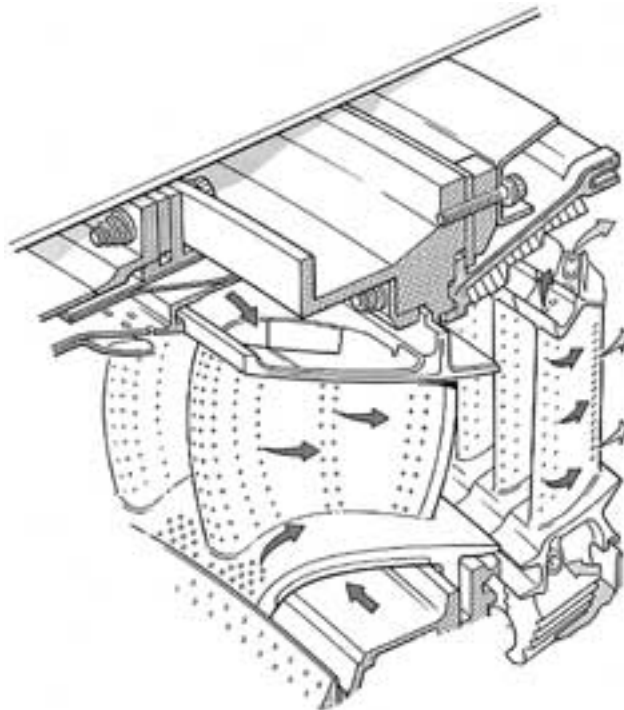
a) For a turbojet engine with a compressor pressure ratio of 25, and a turbine inlet temperature of 1800K, under what flight conditions (Mach number) would the compressor inlet temperature limit be exceeded? Consider performance at sea-level (Tatm=300K) and at 10km (Tatm = 216K)

b) At a constant flight Mach number of 1.2, how much do the thermal and propulsive efficiency change between altitudes of sea-level and 10km? Explain this behavior.

c) If you were forced to reduce the turbine inlet temperature to 1400K because the turbine durability group was having difficulty designing the blades to withstand temperatures of 1800K, how would it impact the performance of the vehicle (consider M=1.2 and 10km only)?

Unified Engineering II **Spring 2004**

Problem S13 (Signals and Systems)

In class, you learned about a *smoother*, with transfer function

$$G_1(s) = \frac{-a^2}{(s-a)(s+a)}$$

The smoother is an example of a *low-pass filter*, which means that it tends to attenuate high-frequency sine waves, but "pass" low-frequency sine waves. Unfortunately, the smoother is non-causal, which means that it can't be implemented in real time. A similar causal low-pass filter is

$$G_2(s) = \frac{a^2}{(s+a)^2}$$

In this problem, you will compare these two low-pass filters, to see how they affect sinusoidal inputs. Consider an input signal

$$u(t) = \cos \omega t$$

1. Find the transfer function, $G_1(j\omega)$, as a function of frequency, ω.

2. Since the transfer function is complex, it can be represented as

   $$G_1(j\omega) = A_1(\omega)e^{j\phi_1(\omega)}$$

   where the amplitude of the transfer function is $A_1(\omega)$, and the phase of the trasnfer function is $\phi_1(\omega)$. Find $A_1(\omega)$ and $\phi_1(\omega)$.

3. Find the transfer function, $G_2(j\omega)$, as a function of frequency, ω, as well as $A_2(\omega)$ and $\phi_2(\omega)$.

4. For the input $u(t)$ above, show that the output of the system G_1 is

   $$y_1(t) = A_1(\omega)\cos(\omega t + \phi_1(\omega))$$

   and do likewise for system G_2.

5. A_1 and A_2 determine how much the magnitude of the input cosine wave is affected by each filter. Ideally, A_1 and A_2 would be 1, meaning that the filters don't change the magnitude of the input sine at all. Which filter (if either) changes the magnitude the least?

6. ϕ_1 and ϕ_2 determine how much the phase of the input cosine wave is affected by each filter. Non-zero values of ϕ correspond to a shifting left or right (*i.e.*, advancing or delaying) the sine wave. Ideally, ϕ_1 and ϕ_2 would be zero, meaning that the filters don't change the phase of the input sine at all. Which filter (if either) produces the least phase shift?

7. Explain why the non-causal filter is preferred in signal processing applications where it can be applied.