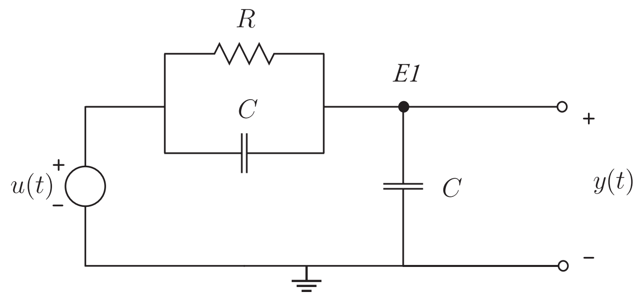
(a)



We can use impedance methods to solve for $Y(s)$ in terms of $U(s)$. Label ground and E_1 as shown. Then KCL at E_1 yields

$$Cs(E_1 - 0) + \left(Cs + \frac{1}{R}\right)(E_1 - U) = 0 \tag{1}$$

Simplifying, we have

$$\left(2Cs + \frac{1}{R}\right) E_1 = \left(Cs + \frac{1}{R}\right) U$$

Since we are finding the step response,

$$U(s) = \frac{1}{s}, \qquad \text{Re}[s] > 0$$

Plugging in numbers, we have

$$(0.5s + 0.5)E_1(s) = (0.25s + 0.5)\,\frac{1}{s}$$

Solving for E_1, we have

$$E_1(s) = \frac{0.25s + 0.5}{(0.5s + 0.5)s} = \frac{0.5s + 1}{s(s+1)}$$

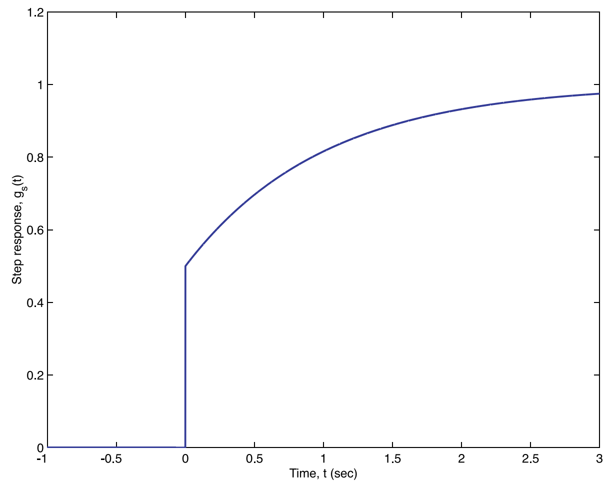
The region of convergence must be $\text{Re}[s] > 0$, since the step response is causal, and the pole at $s = 0$ is the rightmost pole. Using partial fraction expansions,

$$E_1(s) = \frac{1}{s} - \frac{0.5}{s+1}$$

Therefore, $g_s(t) = y(t) = e_1(t)$ is the inverse transform of $E_1(t)$, so

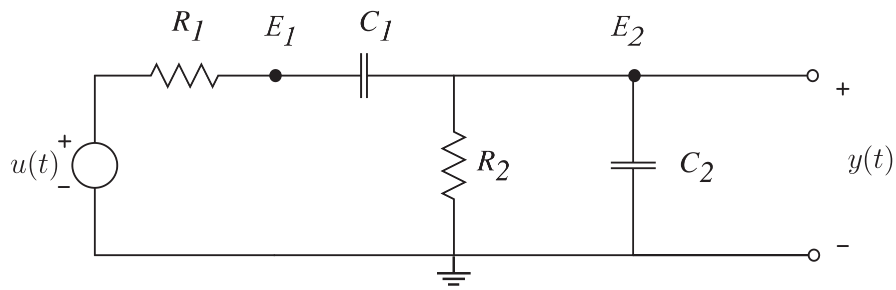
$$y(t) = \left(1 - 0.5e^{-t}\right)\sigma(t)$$

The step response is plotted below:



Normal differential equation methods are difficult to apply, because we cannot apply the normal initial condition that $e_1(0) = 0$. This is because the chain of capacitors running from the voltage source to ground causes there to be an impulse of current at time $t = 0$, and the voltages across the capacitors change instantaneously at $t = 0$. It is possible to use differential equation methods, we just have to be more careful about the initial conditions. However, Laplace methods are easier.

(b)



Again, use impedance methods, using the node labelling above. Then the node equations are

$$\begin{aligned} (C_1 s + G_1)E_1 \quad - \quad & C_1 s E_2 &= G_1 U \\ -C_1 s E_1 \quad + \quad & [(C_1 + C_2)s + G_2]E_2 &= 0 \end{aligned} \tag{2}$$

where $G = 1/R$. We can use Cramer's rule to solve for E_2:

$$\begin{aligned} E_2(s) &= \frac{\begin{vmatrix} C_1 s + G_1 & G_1 U(s) \\ -C_1 s & 0 \end{vmatrix}}{\begin{vmatrix} C_1 s + G_1 & -C_1 s \\ -C_1 s & (C_1 + C_2)s + G_2 \end{vmatrix}} \\ &= \frac{G_1 C_1 s}{C_1 C_2 s^2 + (G_1 C_1 + G_1 C_2 + G_2 C_1)s + G_1 G_2} U(s) \end{aligned} \tag{3}$$

Since we are finding the step response,

$$U(s) = \frac{1}{s}, \qquad \mathrm{Re}[s] > 0$$

Plugging in numbers, we have

$$Y(s) = E_2(s) = \frac{0.1s}{0.06s^2 + 0.35s + 0.25} \frac{1}{s} = \frac{5/3}{s^2 + 5.833\bar{3}s + 4.166\bar{6}} \tag{4}$$

In order to find $y(t)$, we must expand $Y(s)$ in a partial fraction expansion. To do so, we must factor the denominator, using either numerical techniques or the quadratic formula. The result is

$$s^2 + 5.833\bar{3}s + 4.166\bar{6} = (s + 5)(s + 0.833\bar{3}) \tag{5}$$

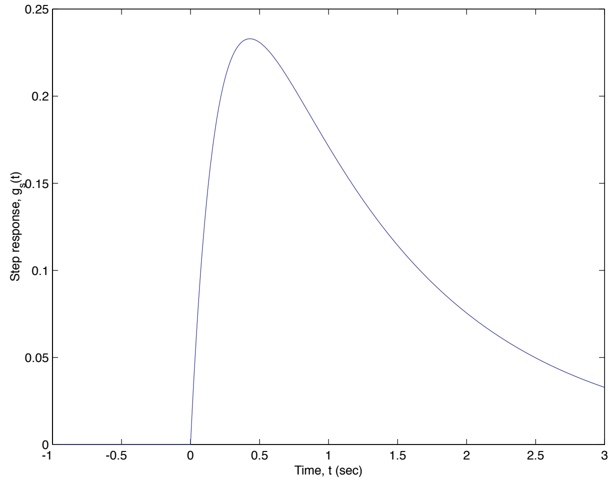
We can use the coverup method to factor $Y(s)$, so that

$$Y(s) = \frac{5/3}{(s + 5)(s + 0.833\bar{3})} = \frac{-0.4}{s + 5} + \frac{0.4}{s + 0.833\bar{3}} \tag{6}$$

The region of convergence must be $\mathrm{Re}[s] > -0.833\bar{3}$, since the step response is causal, and the r.o.c. is to the right of the right-most pole. Therefore, the step response is given by the inverse transform of $Y(s)$, so that

$$g_s(t) = \left(-0.4e^{-5t} + 0.4e^{-0.833\bar{3}t} \right) \sigma(t) \tag{7}$$

The step response is plotted below:


Step response, $g_s(t)$
Time, t (sec)
0.25
0.2
0.15
0.1
0.05
0
-1
-0.5
0
0.5
1
1.5
2
2.5
3