Unified Engineering II **Spring 2004**

Problem S6 (Signals and Systems)

Note: This problem is similar to one given a couple years ago. Please try to do this one without looking at bibles — the solution is instructive.

This problem shows, in a somewhat simplified way, how a radar system determines the distance to an aircraft. More basically, it will give you practice doing convolution using the "flip and slide" method.

The radar sends out a signal, $u(t)$, that reflects off the aircraft and returns to the radar system. The time it takes the signal to return is twice the distance to the aircraft, divided by the speed of light. The received signal is $u(t-T)$, where T is the round trip travel time of the signal.

It may not seem obvious, but it's not easy to determine T by measuring the returned signal. In principal, one could simply display the returned signal on an oscilloscope and measure the time of some key feature of the signal, such as the leading edge of the signal, or the peak of the signal, or whatever. However, the signal received by the radar is relatively weak, because most of the energy radiated by the radar does not strike the aircraft, and most of the reflected energy is not received by the radar. Often the noise in the received signal is as large or larger than the signal itself.

Instead, a better approach is to try to identify the whole received signal instead of a single key feature. How can we do this? Convolution! The received signal is run through an LTI system G with impulse response $g(t)$. The impulse response is designed to accentuate the signal, so that one can measure a key feature of the output signal, $y(t)$, which is relatively insensitive to noise. Often, the best choice for $g(t)$ is a time-reversed version of $u(t)$. That is,

$$g(t) = u(-t) \tag{1}$$

This is called a "matched filter."

1. For the signal $u(t)$ shown below, and $g(t) = u(-t)$, find the convolution

$$y(t) = g(t) * u(t)$$

   using the flip and slide method.

2. Using the results of part (1) and the LTI properties of the system G, what is the convolution

$$y(t) = g(t) * u(t-T)$$

3. What feature of $y(t)$ would you use to identify the time T?

4. Can you explain why the right impulse response for G is a signal that has the same shape as $u(t)$, but time reversed?