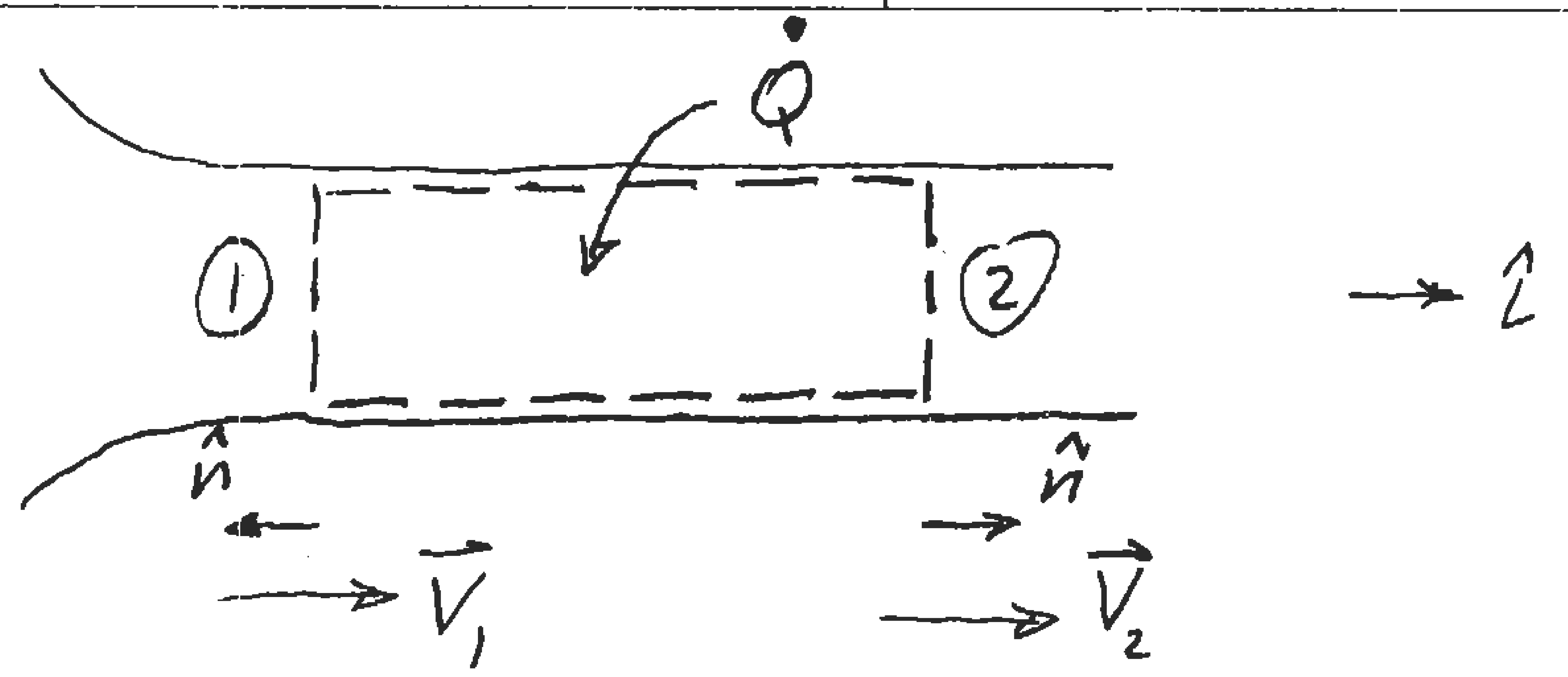
Control Volume



Mass: $\oint \rho \vec{V}\cdot\hat{n}\, dA = 0 \Rightarrow -\rho_1 V_1 A + \rho_2 V_2 A = 0$

or $\boxed{\rho_1 V_1 = \rho_2 V_2}$ (1)

force on radiator

Momentum: $\oint \rho(\vec{V}\cdot\hat{n})\vec{V}\, dA + \oint p\hat{n}\, dA + \vec{R} = 0$

$\Rightarrow -\rho_1 V_1 A V_1 \hat{\imath} + \rho_2 V_2 A V_2 \hat{\imath} - p_1 A \hat{\imath} + p_2 A \hat{\imath} + R\hat{\imath} = 0$

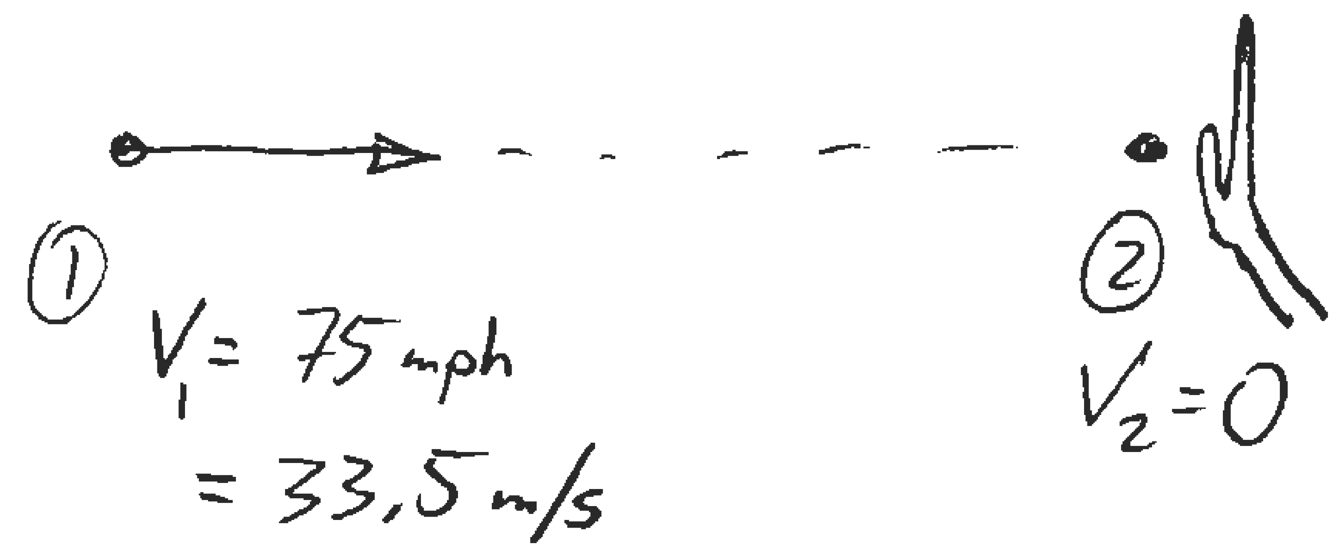
or $\boxed{\rho_1 V_1^2 + p_1 = \rho_2 V_2^2 + p_2 + \cancel{R} \quad \text{neglect as given}}$ (2)

Energy: $\oint \rho(\vec{V}\cdot\hat{n}) h_0\, dA = \overbrace{\iiint \rho \dot{q}\, d\mathcal{V}}^{\dot{Q}} + \iiint \rho \cancelto{0}{\vec{g}\cdot\vec{V}}\, d\mathcal{V}$

$-\rho_1 V_1 A h_{0_1} + \rho_2 V_2 A h_{0_2} = \dot{Q}$

or $\boxed{\rho_1 V_1 h_{0_1} + \dot{Q}/A = \rho_2 V_2 h_{0_2}}$ (3)

Adiabatic (no heat) + Reversible process (frictionless) $\longrightarrow$ Isentropic



$$h_2 = h_{o_2} = h_{o_1} = c_p T_1 + \tfrac{1}{2} V_1^2 = 1004 \text{ J/kg°K} \cdot 300\text{K°} + \tfrac{1}{2}\, 33.5^2 \text{ m}^2/\text{s}^2$$

$$h_2 = 301761.1 \text{ J/kg}$$

$$\boxed{\begin{array}{l} T_2 = h_2/c_p = 300.56 \text{ K°} \\ \Delta T = 0.56 \text{ K°} \end{array}}$$

$$p_2 = p_{o_2} = p_{o_1} = p_1 \left[1 - \frac{V_1^2}{2h_{o_1}}\right]^{-3.5} = p_1 \cdot 1.00654$$

$$\boxed{\begin{array}{l} p_2 = 1.00654 \times 10^5 \text{ Pa} \\ \Delta p = 654 \text{ Pa} \approx \tfrac{1}{2}\rho V^2 \text{ (low speed)} \end{array}}$$

OK to use Bernoulli here

$$\rho_2 = \rho_{o_2} = \rho_{o_1} = \rho_1 \left[1 - \frac{V_1^2}{2h_{o_1}}\right]^{-2.5} = \rho_1 \cdot 1.00466$$

$$\boxed{\rho_2 = 1.2056 \text{ kg/m}^2}$$

Note: Data as given doesn't <u>exactly</u> satisfy state equation. Some numerical differences will occur if the state equation is used instead of one of the adiabatic or isentropic relations.

a) $M_\infty = \dfrac{V_\infty}{a_\infty} = \dfrac{V_\infty}{\sqrt{\gamma R T_\infty}}$, but $RT = \dfrac{p}{\rho}$, so $a_\infty = \sqrt{\dfrac{\gamma p_\infty}{\rho_\infty}}$

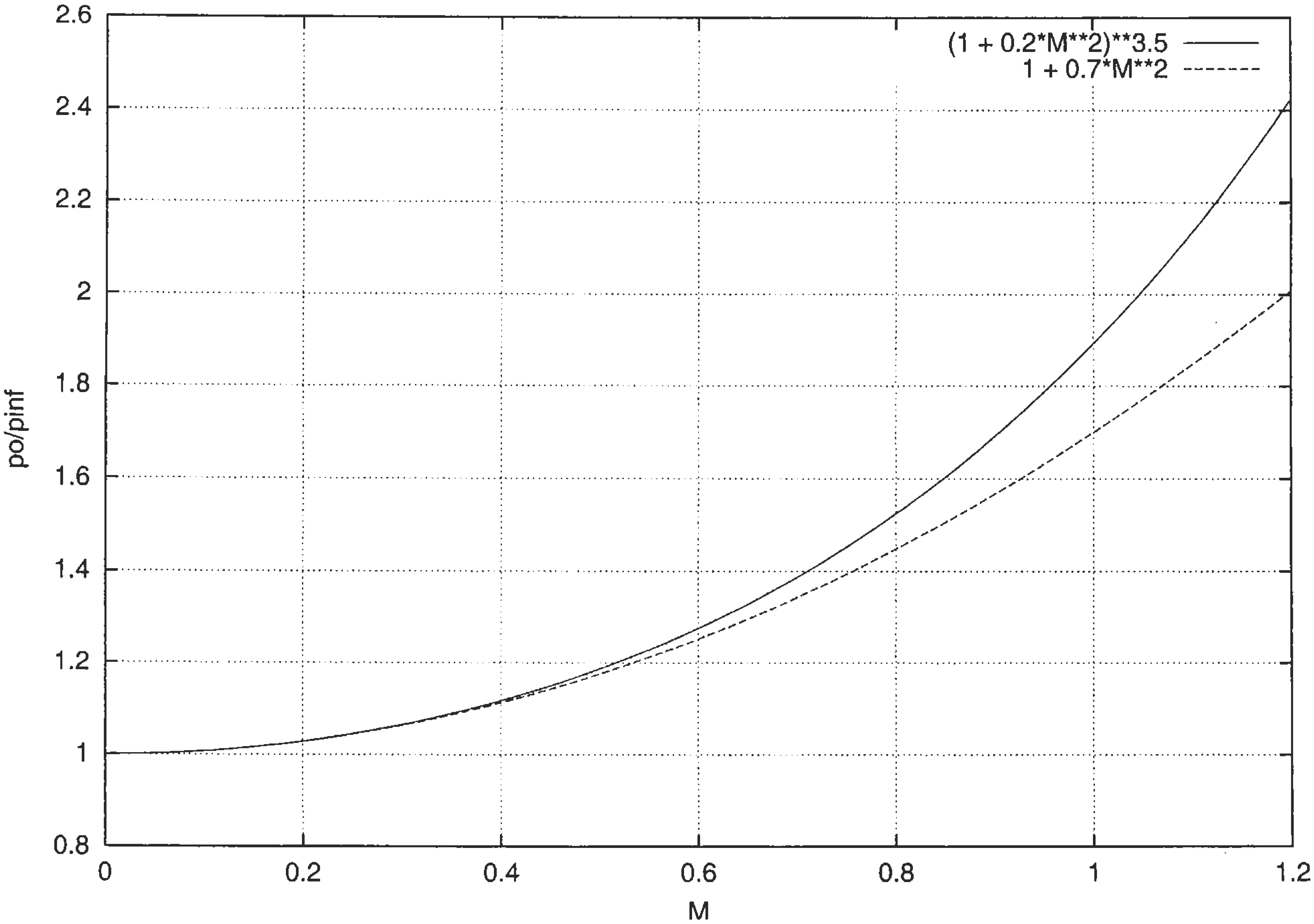
$$\boxed{M_\infty = V_\infty \sqrt{\frac{\rho_\infty}{\gamma p_\infty}}}$$

b) $$\boxed{p_0 = p_\infty \left[1 + \frac{\gamma - 1}{2} M_\infty^2\right]^{\frac{\gamma}{\gamma-1}}}$$ exact

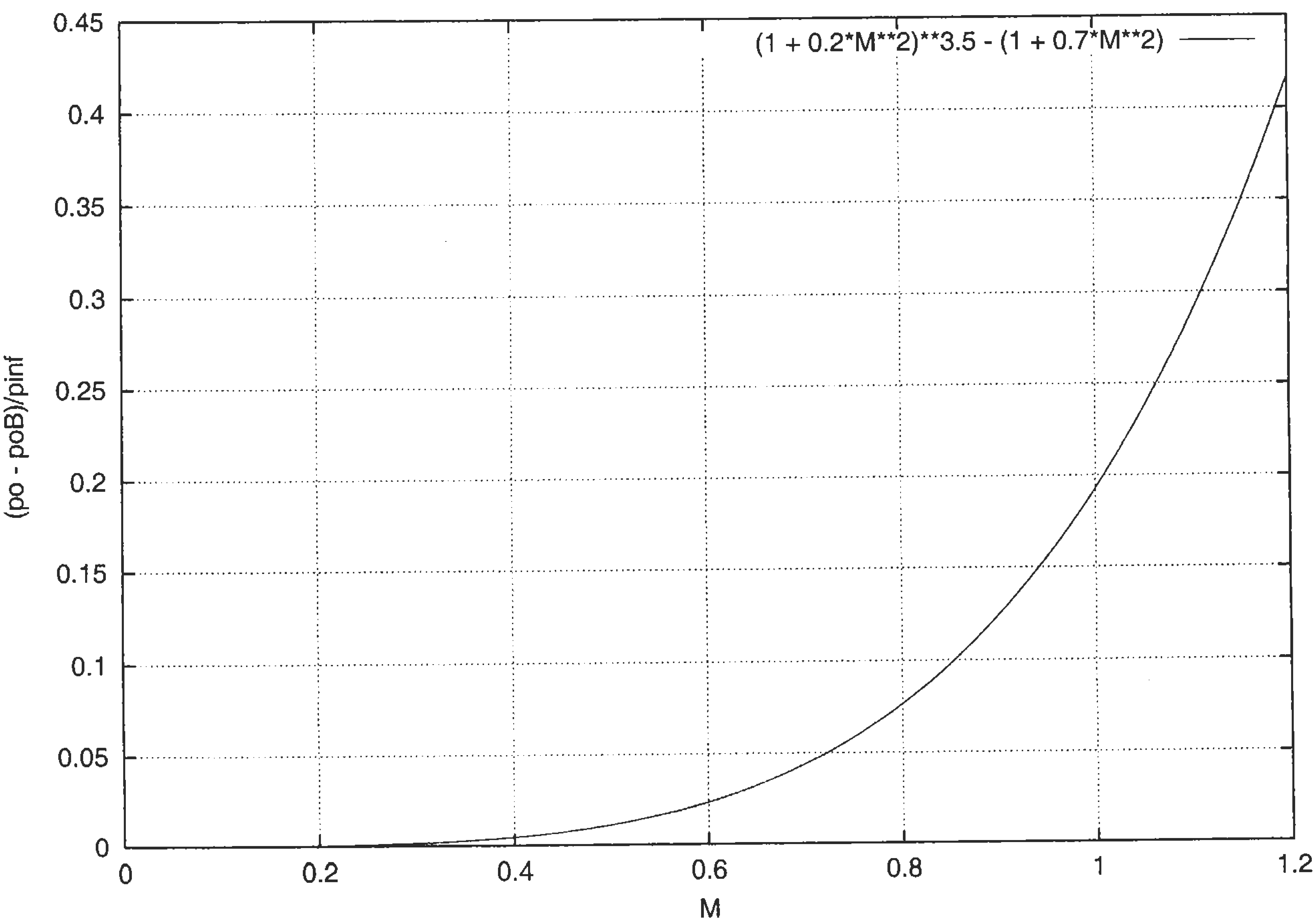
$$p_0 = p_\infty + \frac{1}{2}\rho_\infty V_\infty^2$$
$$= p_\infty + \frac{1}{2}\rho_\infty M_\infty^2 \cdot \frac{\gamma p_\infty}{\rho_\infty}$$
$$\boxed{p_0 = p_\infty\left[1 + \frac{\gamma}{2} M_\infty^2\right]}$$ Bernoulli

Plot $\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}$ and $1 + \frac{\gamma}{2} M_\infty^2$ attached

Plot $\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}} - \left(1 + \frac{\gamma}{2} M_\infty^2\right)$ attached



(1 + 0.2*M**2)**3.5
1 + 0.7*M**2
po/pinf
M
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0
0.2
0.4
0.6
0.8
1
1.2



(1 + 0.2*M**2)**3.5 - (1 + 0.7*M**2)
(po - poB)/pinf
M
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
1.2

For any taper r: $\quad c(y) = c_{avg}\,\frac{2}{1+r}\left[1-(1-r)\frac{2y}{b}\right]$

a) Assuming $q \sim c$: $\quad \boxed{q(y) = q_{avg}\,\frac{2}{1+r}\left[1-(1-r)\frac{2y}{b}\right]}$

Total lift on half span: $F = q_{avg}\cdot\frac{b}{2} = 10\,N \;\rightarrow\; \boxed{q_{avg} = \frac{10N}{1m} = 10\,N/m}$

$$\boxed{S(y) = \int_{b/2}^{y} q(y)\,dy = q_{avg}\frac{2}{1+r}\left[y-(1-r)\frac{y^2}{b}\right]_{b/2}^{y} = q_{avg}\frac{2}{1+r}\left[y-\frac{b}{2}+\frac{1-r}{b}\left(\frac{b^2}{4}-y^2\right)\right]}$$

$$M(y) = \int_{b/2}^{y} S(y)\,dy = q_{avg}\frac{2}{1+r}\left[\frac{1}{2}y^2-\frac{b}{2}y+(1-r)\left(\frac{b^2}{4}y-\frac{1}{3}y^3\right)\right]_{b/2}^{y}$$

$$\boxed{M(y) = q_{avg}\frac{2}{1+r}\left[\frac{1}{2}\left(y^2-\frac{b^2}{4}\right)+\frac{b}{2}\left(\frac{b}{2}-y\right)+\left(\frac{1-r}{b}\right)\left(\frac{b^2}{4}\left(y-\frac{b}{2}\right)+\frac{1}{3}\left(\frac{b^3}{8}-y^3\right)\right)\right]}$$

could simplify this I suppose. Plots attached

b) $M = Ph = Pc\tau \;\rightarrow\; P(y) = \pm\frac{M(y)}{c(y)\,\tau}$, Plots attached.

c) $P = A\sigma \;\rightarrow\; A_{min}(y) = \frac{P(y)}{\sigma_{max}}$ same plot as $P(y)$, aside from scale

Area is roughly parabolic. $\quad Vol = \int_0^{b/2} A(y)\,dy \approx \frac{1}{3}A(0)\cdot\frac{b}{2}$

$$Vol \simeq \frac{1}{3}\frac{P(0)}{\sigma_{max}}\cdot\frac{b}{2} = \frac{b}{6}\frac{1}{\sigma_{max}}\frac{M(0)}{c(0)}\frac{1}{\tau} \quad \text{(one cap for half-wing)}$$

we have $M(0) = q_{avg}\frac{2}{1+r}\left[-\frac{b^2}{8}+\frac{b^2}{4}+\frac{1-r}{b}\left(-\frac{b^3}{8}+\frac{b^3}{24}\right)\right] = q_{avg}\frac{2}{1+r}\left[\frac{b^2}{8}-(1-r)\frac{b^2}{12}\right]$

$c(0) = c_{avg}\frac{2}{1+r}$

$$\therefore\; Vol = \frac{b^3}{6}\frac{1}{\sigma_{max}\tau}\frac{q_{avg}}{c_{avg}}\left(\frac{1}{8}-\frac{1-r}{12}\right) = \begin{cases} 11.9\times10^{-6}\,m^3 = 11.9\,cm^3 & (r=1.0) \\ 7.9\times10^{-6}\,m^3 = 7.9\,cm^3 & (r=0.5)\end{cases}$$

4-cap mass $m = 4\rho\cdot Vol = \begin{cases} 6.0\,g & (r=1.0) \\ 4.0\,g & (r=0.5)\end{cases}$

d) $I = \frac{1}{2}Ah^2 = \frac{1}{2}Ac^2\tau^2 = \frac{1}{2}\frac{M}{c\tau\sigma_{max}}c^2\tau^2 = \frac{1}{2}\frac{Mc\tau}{\sigma_{max}}$

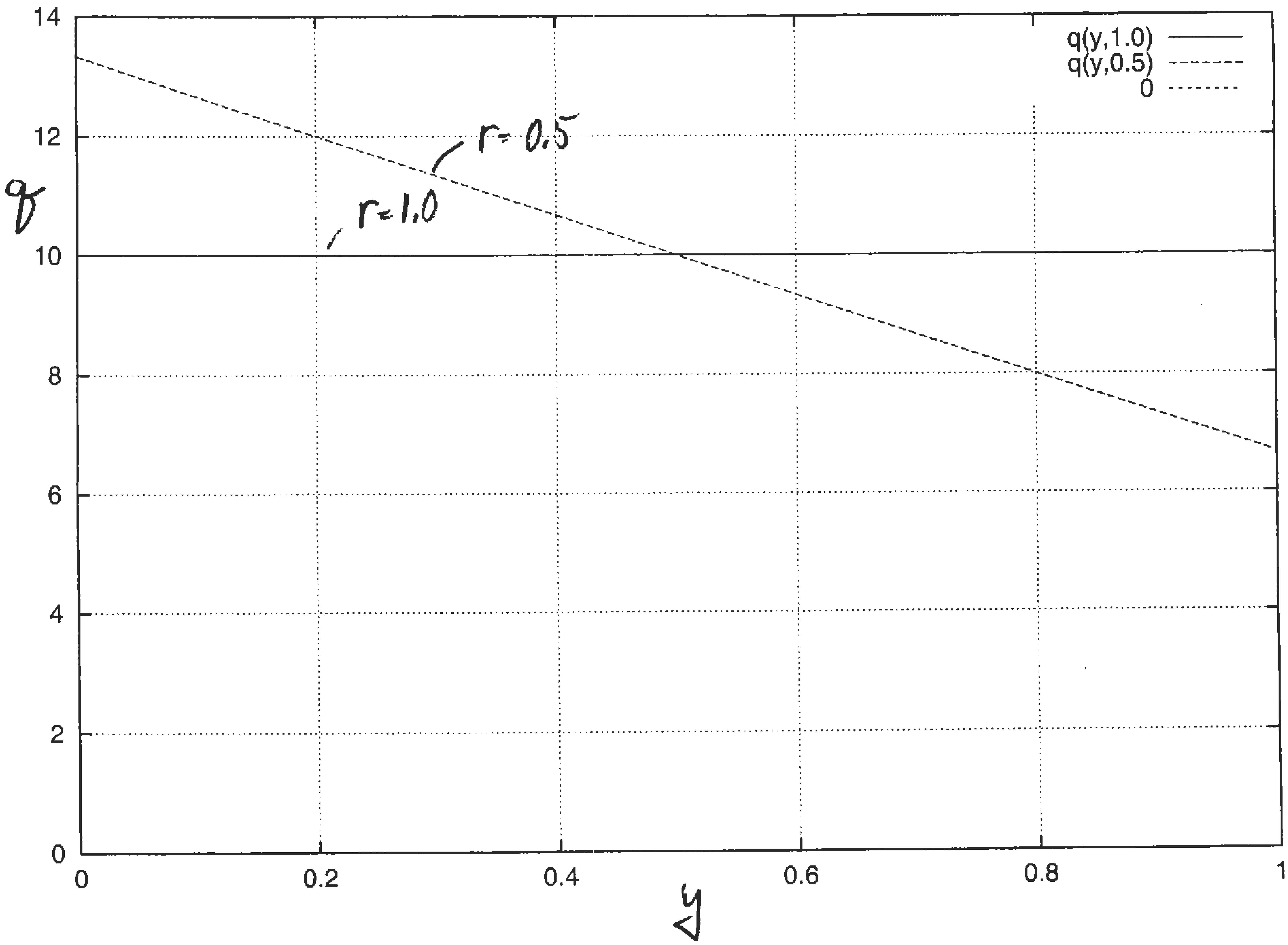
$K = \frac{M}{EI} = \frac{2\sigma_{max}}{E}\frac{1}{c\tau}$, $\quad K(0) = 2\,\frac{7\,MPa}{1.36\,GPa}\cdot\frac{1}{0.08}\,\frac{1}{c(0)} = 0.129\cdot\frac{1+r}{2c_{avg}} = \begin{cases} 0.52/m & r=1.0 \\ 0.39/m & r=0.5\end{cases}$

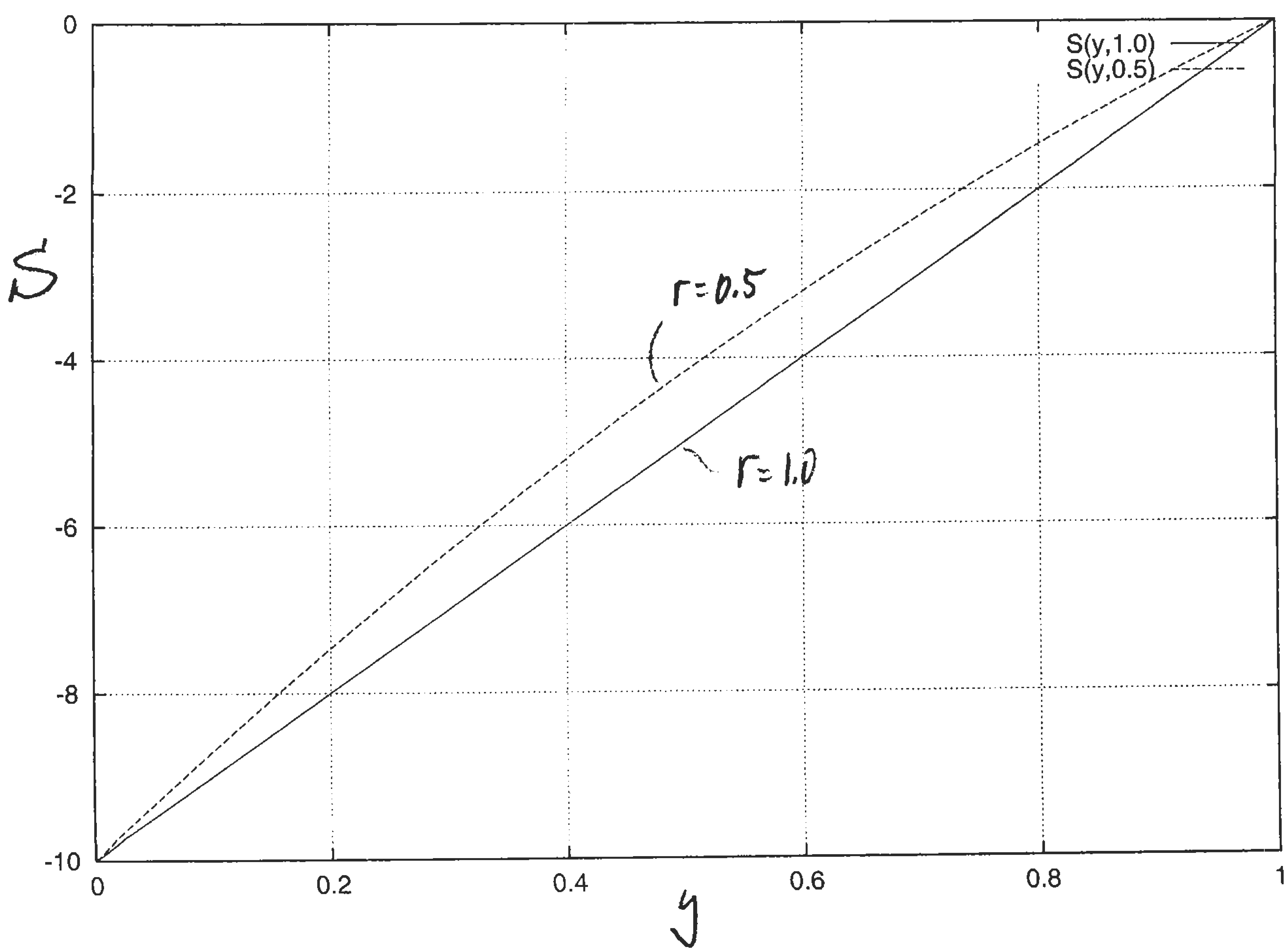
$\delta = \frac{1}{2}K\,(b/2)^2 = \begin{cases} 0.258\,m & r=1.0 \\ 0.193\,m & r=0.5\end{cases}$

The tapered wing seems better in all respects.
- better e (lower C_{D_i})
- lighter spar
- smaller δ

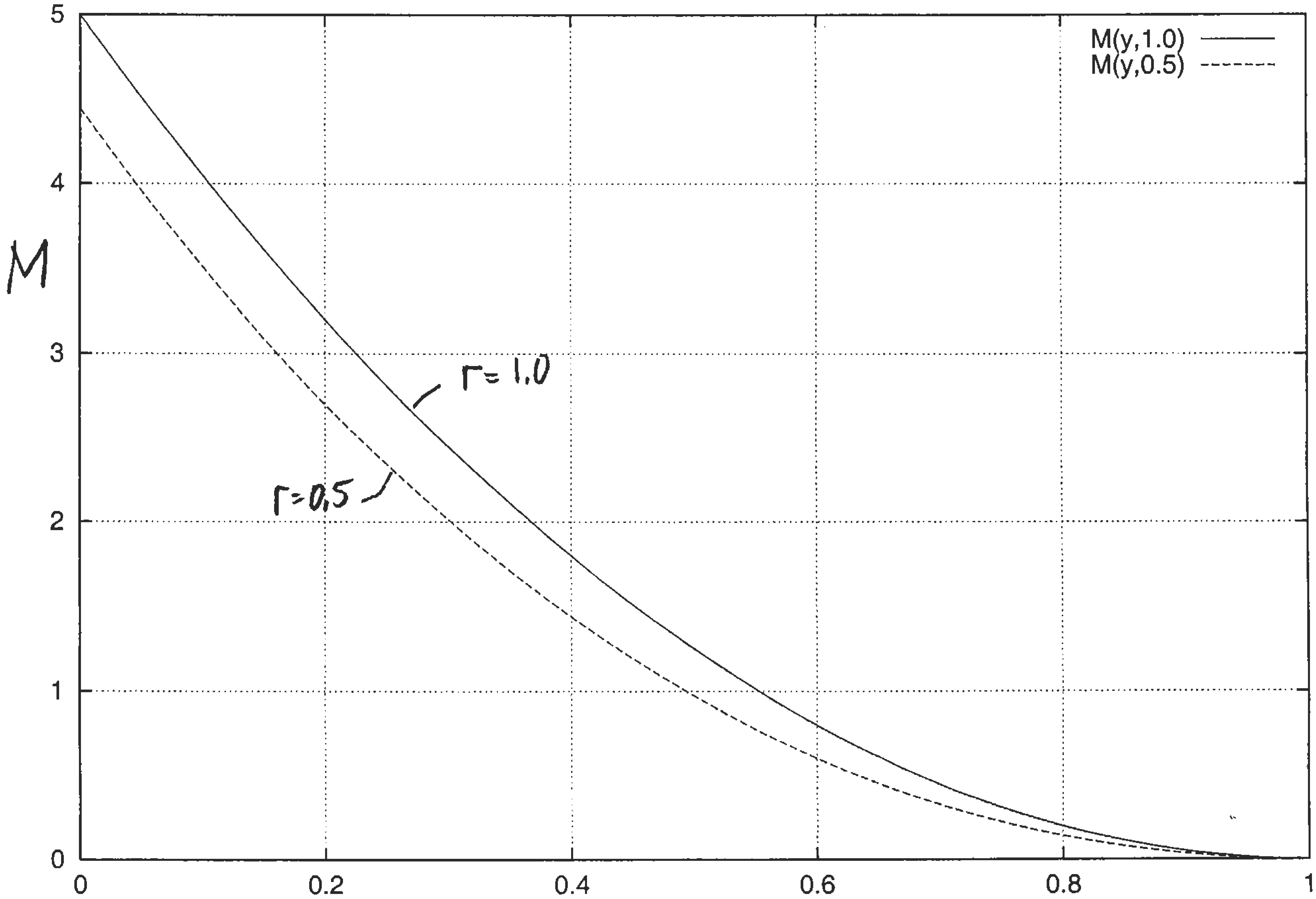
Also, balsa caps are very light. Look attractive.

M10

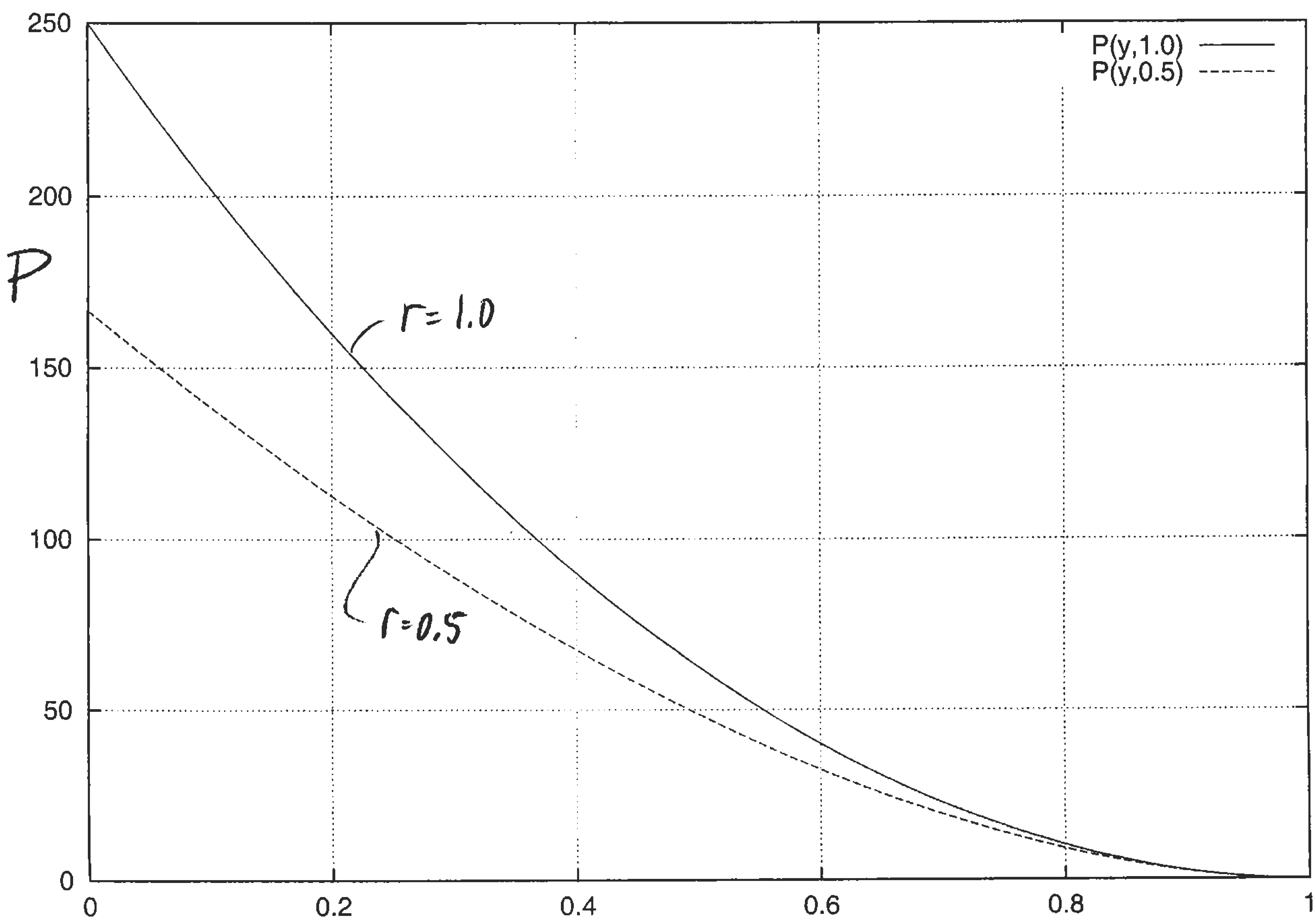




M10



M
M(y,1.0)
M(y,0.5)
r=1.0
r=0.5
0
0.2
0.4
0.6
0.8
1
2
3
4
5



P
P(y,1.0)
P(y,0.5)
r=1.0
r=0.5
0
50
100
150
200
250
0.2
0.4
0.6
0.8
1

M11 Torsion of circular cross-section shafts

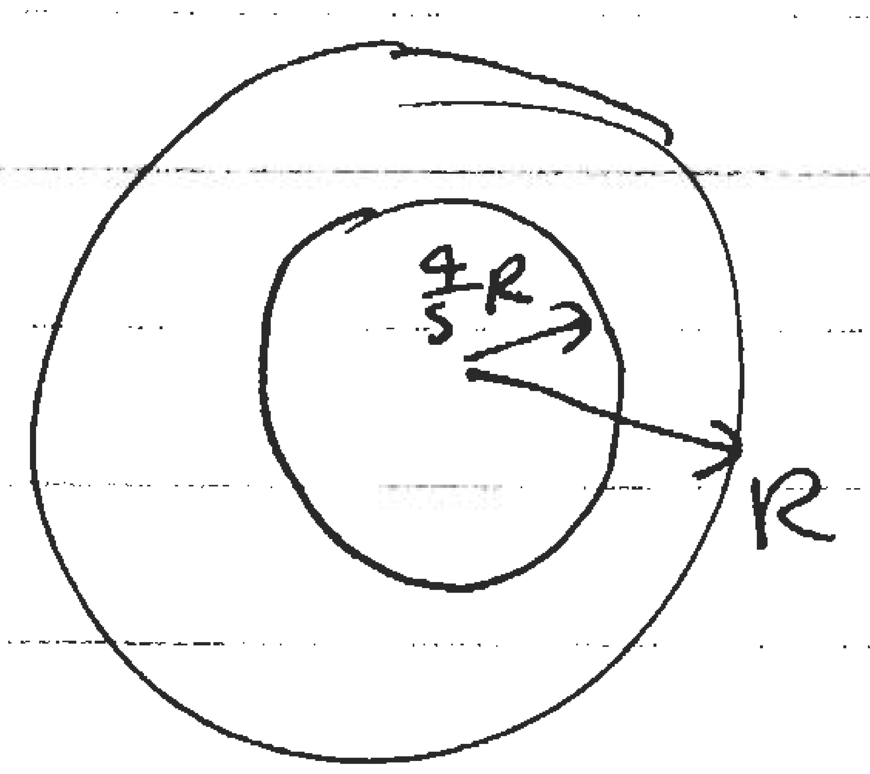
a) Torque shear stress relation: $\tau = \dfrac{Tr}{J}$

for solid circular cross section $J = \dfrac{\pi R^4}{2}$

$$\therefore \quad R^{\cancel{4}\,3} = \frac{2T\cancel{R}}{\pi \tau_y} \Rightarrow R = \sqrt[3]{\frac{2T}{\pi \tau_y}} = \sqrt[3]{\frac{2 \times 200 \times 10^3}{\pi \times 200 \times 10^6}}$$

$$= 0.086\,\text{m} \Rightarrow \text{diameter} = 0.172\,\text{m}$$

b) Hollow shaft



$$J = \frac{\pi R^4}{2} - \frac{\pi}{2}\left(\frac{4}{5}\right)^4 R^4$$

$$= \frac{\pi R^4}{2}\left(1 - \left(\frac{4}{5}\right)^4\right)$$

$$= \frac{\pi R^4}{2}\left(1 - \frac{256}{625}\right)$$

$$\therefore \quad \frac{\pi R^{\cancel{4}\,3}}{2}\left(0.5904\right) = \frac{T\cancel{R}}{\tau_y}$$

$$R = \sqrt[3]{\frac{2T}{\pi(0.5904)\tau_y}} = \sqrt[3]{\frac{2 \times 200 \times 10^3}{\pi \times 0.5904 \times 200 \times 10^6}} = 0.103\,\text{m}$$

$\therefore$ diameter = 0.205 m.

$$\text{Fractional weight} = \frac{\cancel{\pi}\left(0.086^2 - (0.103)^2\left(1-\left(\frac{4}{5}\right)^2\right)\right)}{\cancel{\pi}\,(0.086)^2}$$

$$= 1 - \left(\frac{0.103}{0.086}\right)^2\left(1-\left(\frac{4}{5}\right)^2\right)$$

$$= 0.484 \quad = 48.4\% \text{ weight saving.}$$

c) Twist angle. from $T = GJ\frac{d\phi}{dx}$

Since Torque is constant, twist angle $= L\frac{d\phi}{dx}$

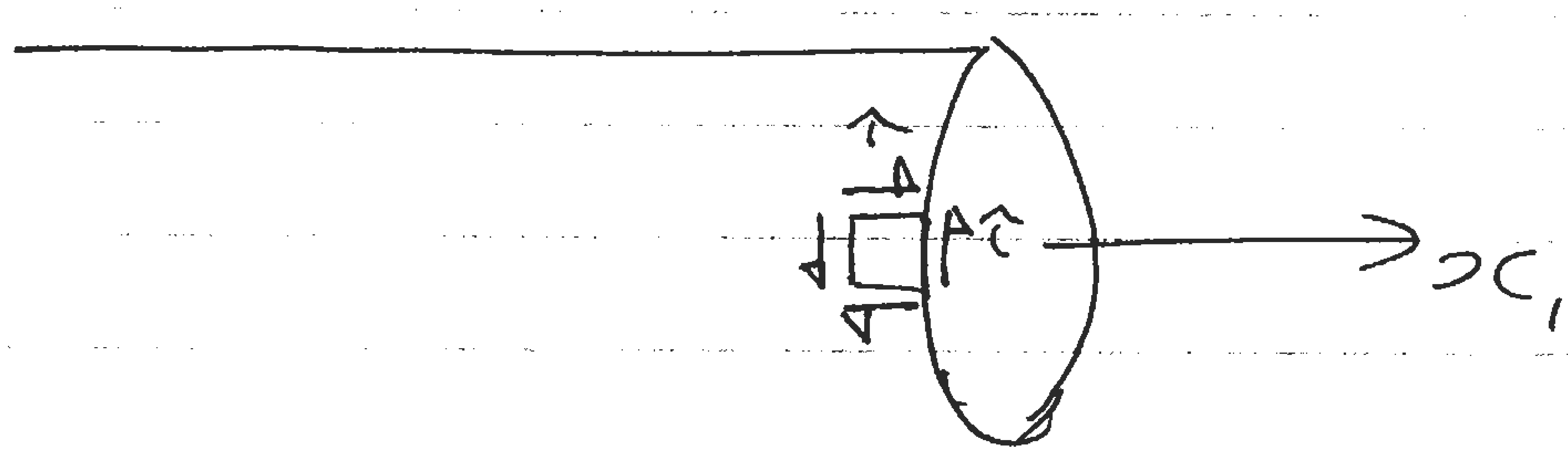
$\therefore$ ratio of twist angles is ratio of J

$$= \frac{\pi(0.103)^4}{x}\left(1-\frac{256}{625}\right) \times \frac{x}{\pi(0.086)^4}\Bigg($$

$$= \frac{(0.103)^4}{(0.086)^4}\left(1-\frac{256}{625}\right) = 1.215$$

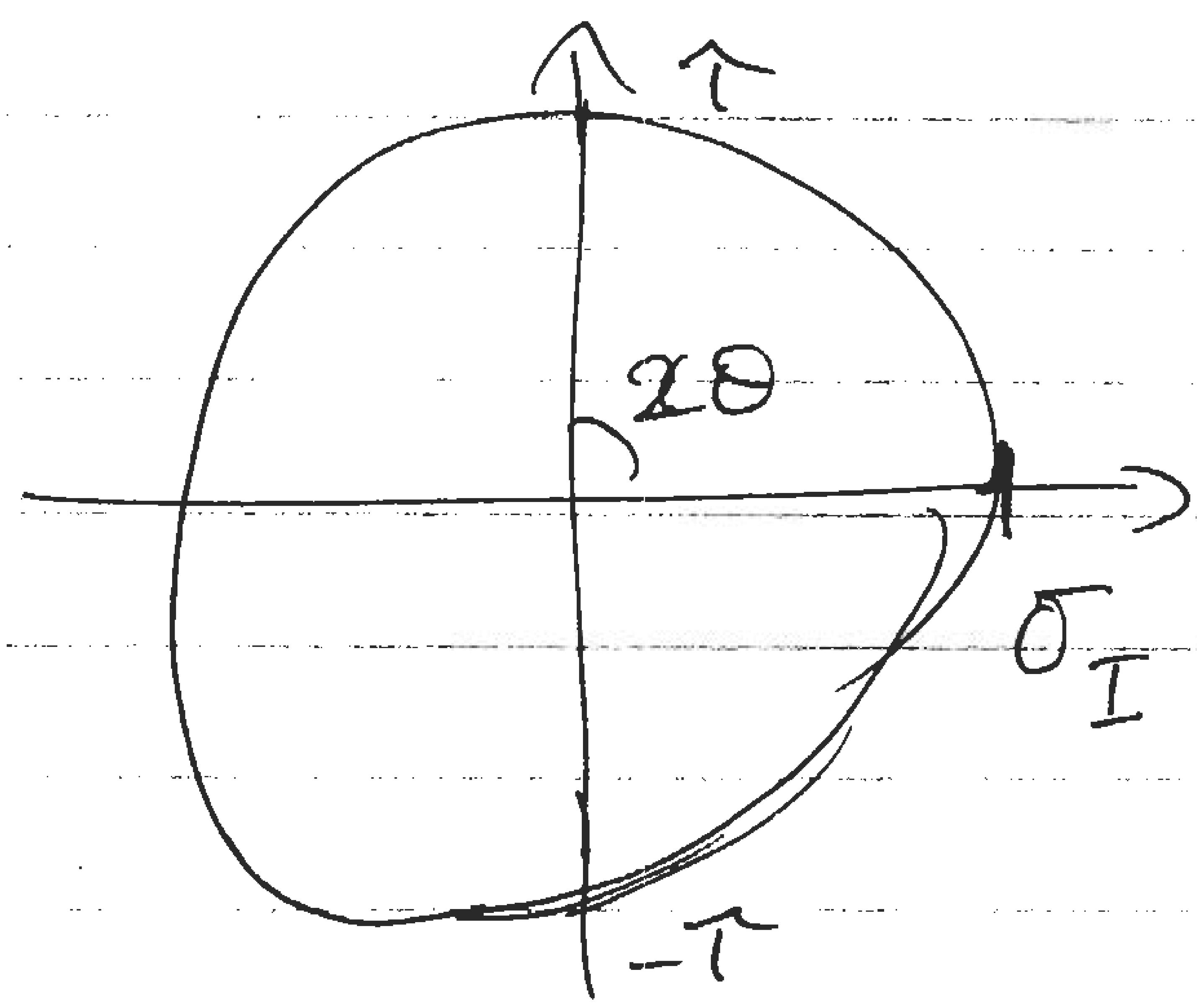
ie. shaft (a) will twist 21.5% more than shaft (b).

d.

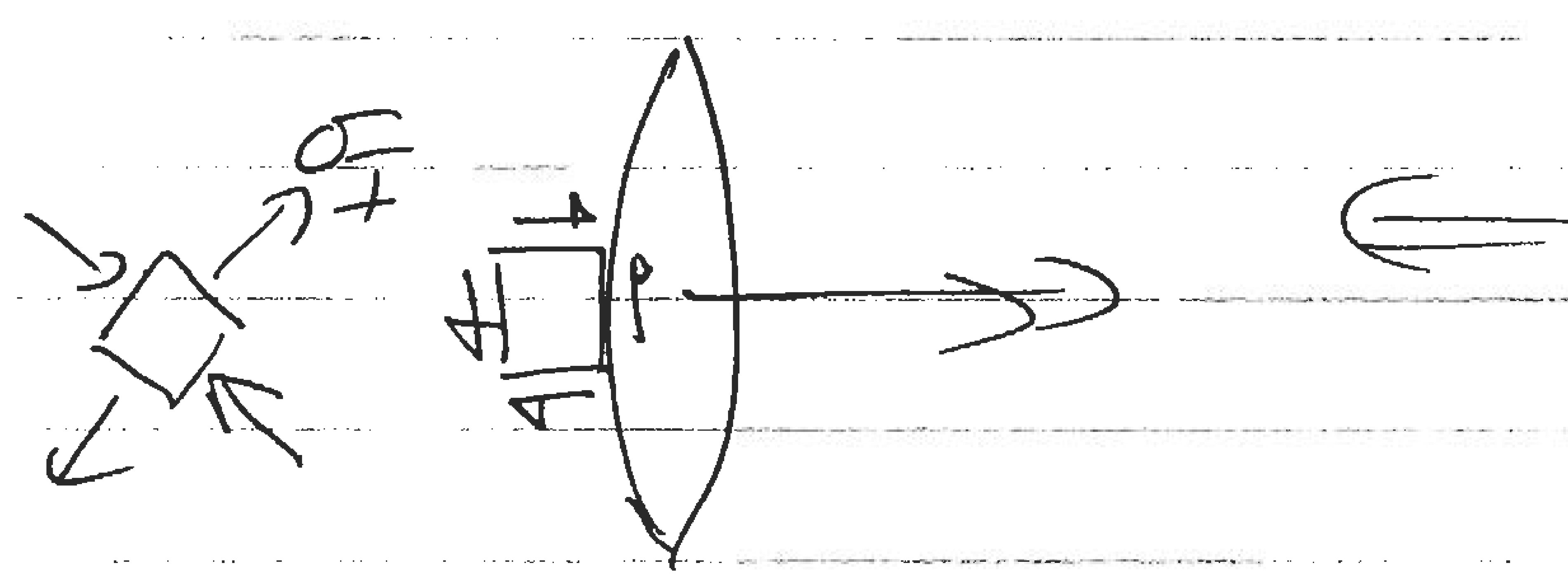


in case b (or a) shear stress τ acts in plane of section.

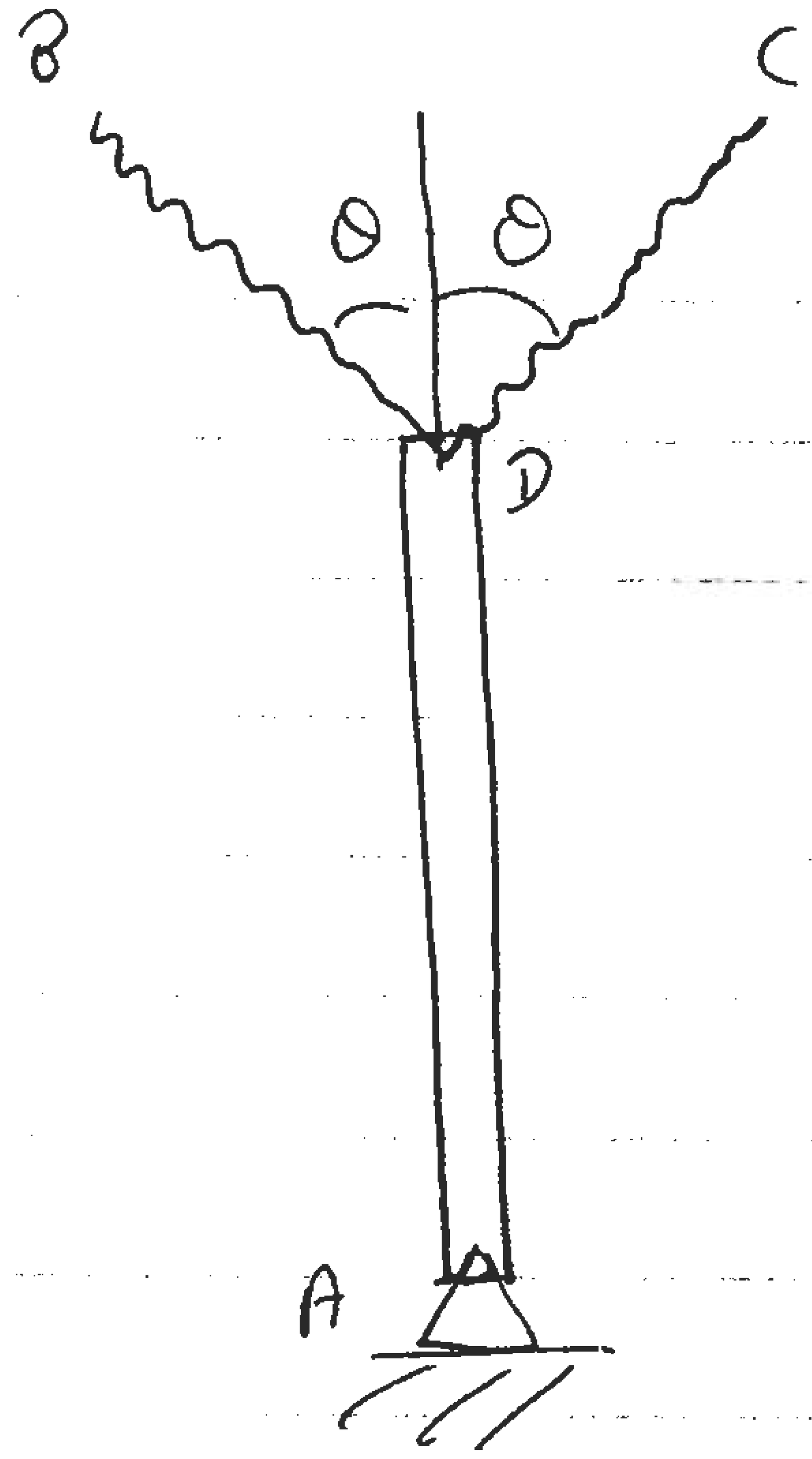
Drawing Mohrs circle



max tensile stress, σ_I will act at 45° to max shear ie.

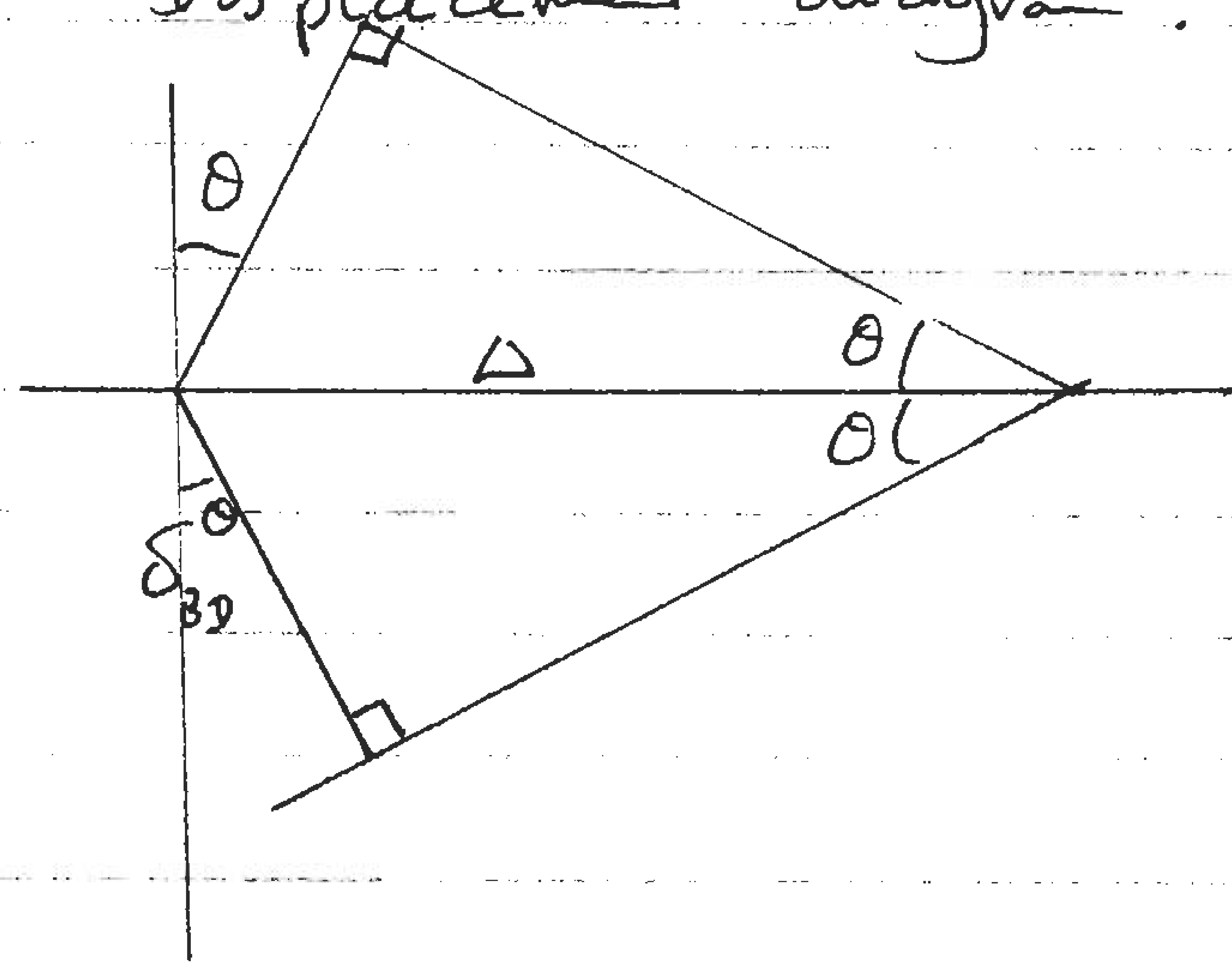


M12



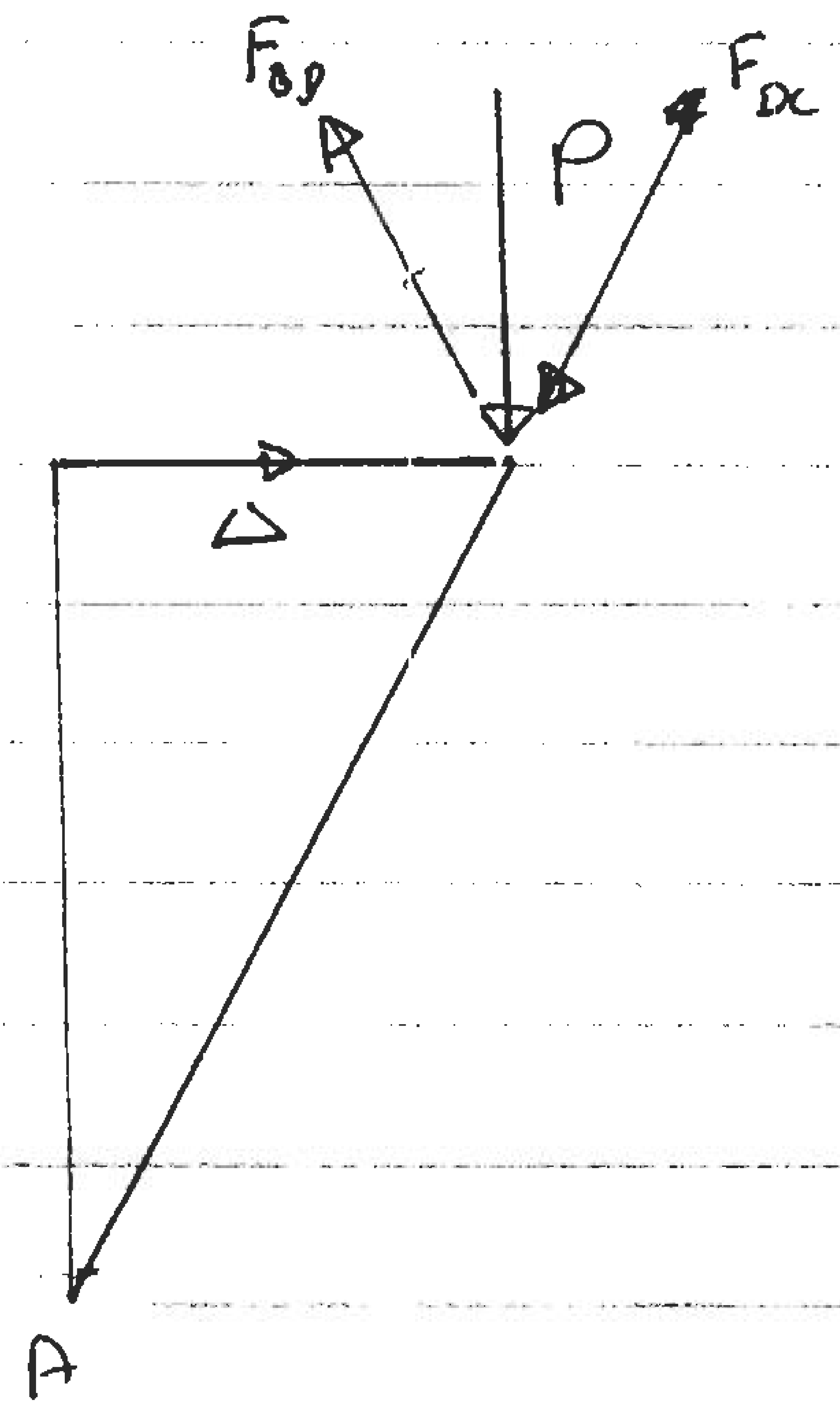
Displace D horizontally by Δ
BD & DC extend by δ and rotate

Displacement diagram.



∴ compatibility: $\Delta \sin\theta = \delta_{BD} = \delta_{DC} = \delta$

BD extending, DC shortening ∴ FBD



$F_{BD} = k\,\delta_{BD} = k\delta$

$F_{DC} = k\,\delta_{DC} = k\delta$

∴ ~~vertical comp:~~ Net vertical force $= 0$

Horizontal component $= 2F_{BD} = 2k\delta$
$= 2k\Delta \sin\theta \Leftarrow$

$\therefore$ equilibrium of moments about A

$$\Rightarrow \circlearrowleft M_A = 0 : \; +2K\cancel{L}\sin\theta . L - P\cancel{L} = 0$$

$$P = 2K \sin\theta L$$

if $P > 2K\sin\theta L$ then collapse occurs $\Leftarrow$

M13 Need to consider possibility of buckling in compressive members of truss

Material selection

achieve certain (required) buckling load while minimizing mass

$$P_{crit} = \frac{\pi^2 E I}{L^2} \qquad \text{~~mass~~ mass} = \pi R^2 L \rho$$

assume circular cross-section $\therefore I = \frac{\pi R^4}{4}$

~~mass~~ rewrite $R = \sqrt{\frac{M}{\pi L \rho}}$ & $R^4 = \left(\frac{M}{\pi L \rho}\right)^2$

$$\therefore P_{crit} = \frac{\pi^{\cancel{2}} E}{L^2} \cdot \frac{\pi}{4} \cdot \left(\frac{M}{\cancel{\pi} L \rho}\right)^2 = \underbrace{M^2}_{F} \quad \underbrace{\cancel{E}\,\frac{\pi}{4L^4}}_{G} \quad \underbrace{\frac{E}{\rho^2}}_{M}$$

Maximize E/ρ^2 for highest buckling load for given mass

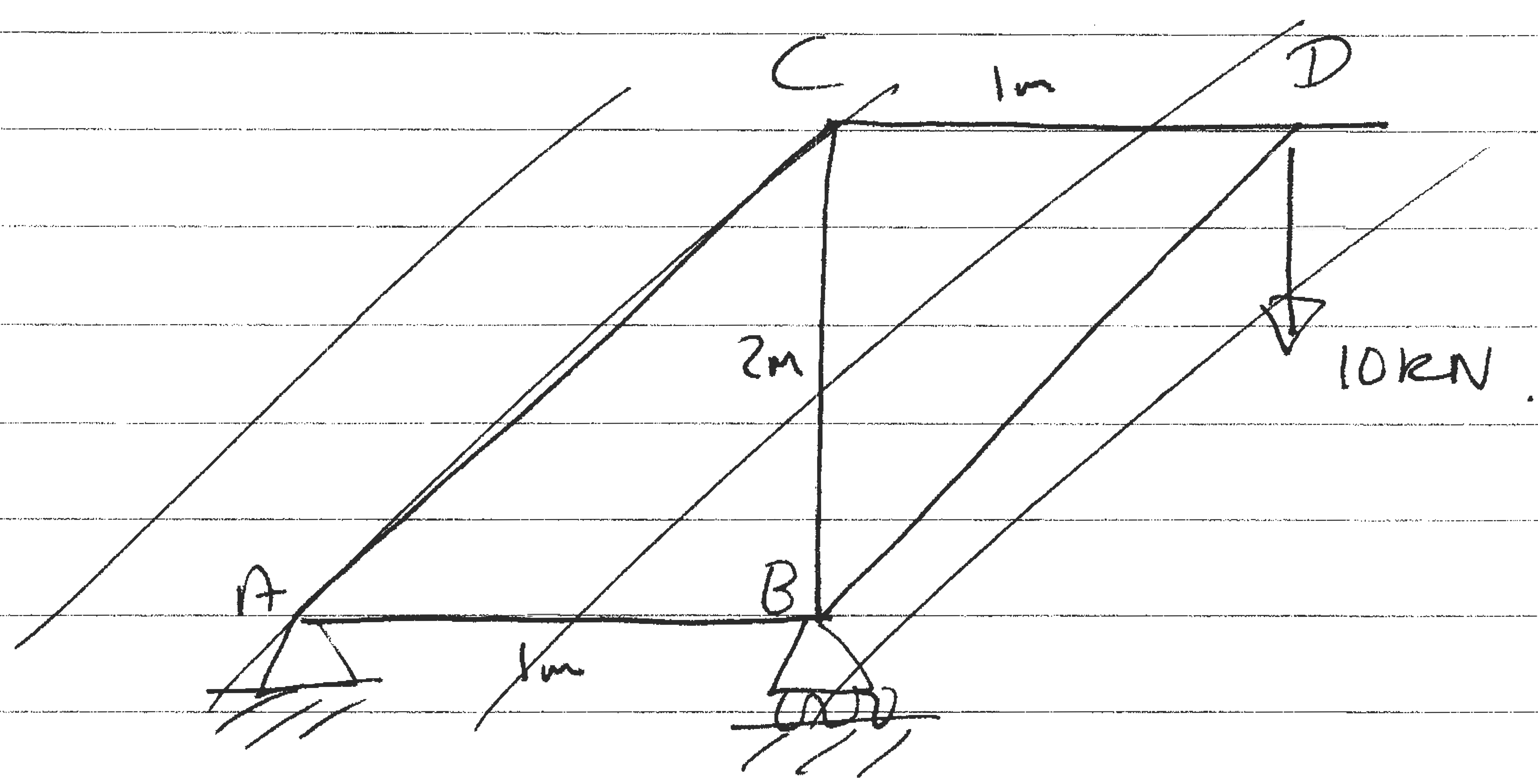
(previously maximize σ_f/ρ)

Reranking materials

	σ_f/ρ	E/ρ^2
Steel	28×10^3	3.3×10^3
Al	125×10^3	9.0×10^3
Ti	188×10^3	5.9×10^3
CFRP	466×10^3	31×10^3
Wood	50×10^3	33×10^3
SiC	100×10^3	45×10^3

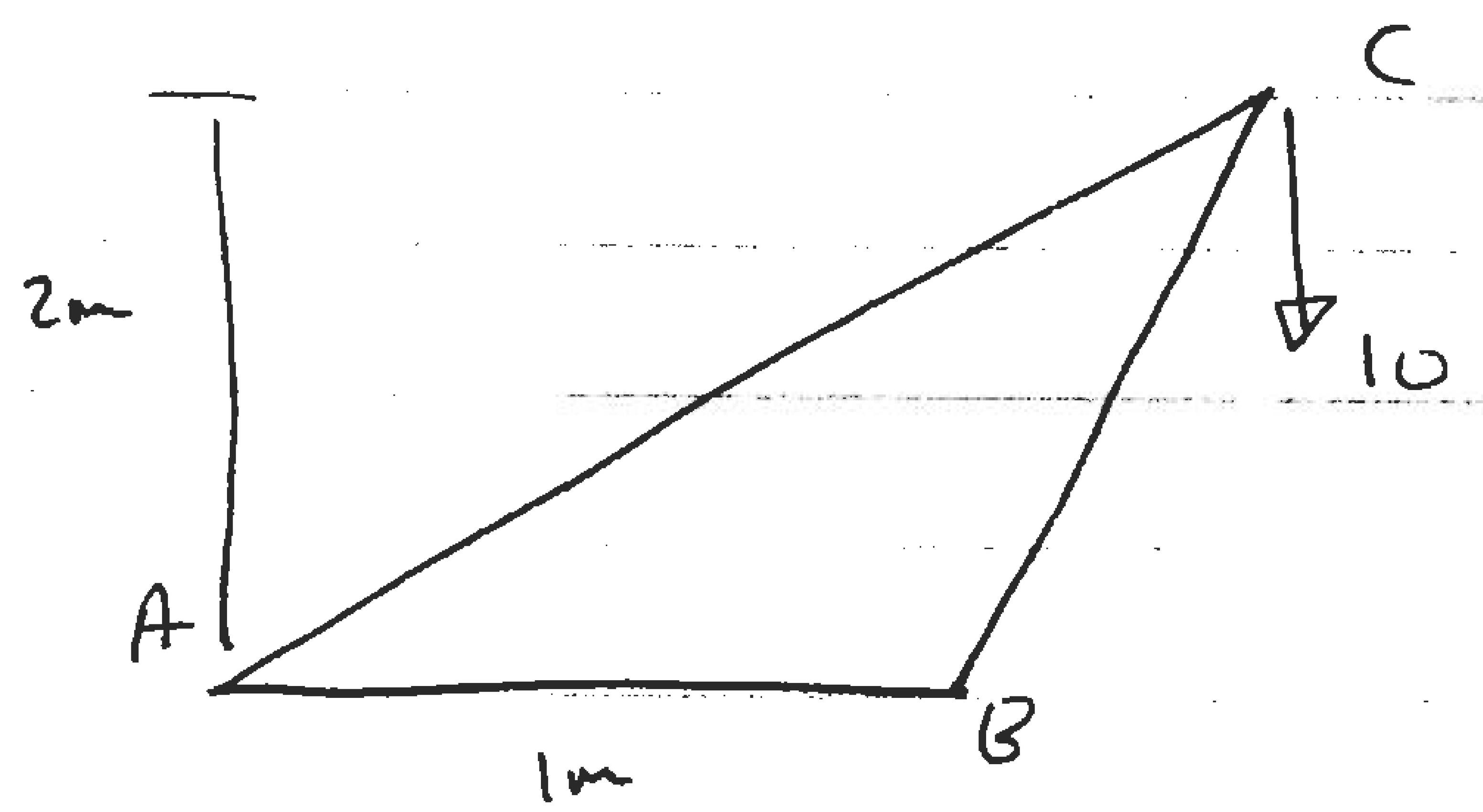
CFRP still looks very good. - high E/ρ^2
Wood might be better in buckling dominated design.

Reconsider design



Costs

Reconsider design.



$F_{AB} = -10\ kN.$

$F_{BC} = -22.4\ kN.$

BC is the longest member at highest compressive force
∴ only need to consider this.

Assume that it is a simply supported column.

$$P_{crit} = \frac{\pi^2 E I}{L^2}$$

given circular cross-section

$$I = \frac{\pi R^4}{4}$$

$$\therefore \quad P_{crit} = \frac{\pi^2 E \pi R^4}{4 L^2} \qquad L = \sqrt{5}\ m$$

$$R = \sqrt[4]{\frac{P_{crit} \times 4 L^2}{\pi^3 E}} = \sqrt[4]{\frac{22.4 \times 10^3 \times 4 \times 5}{\pi^3 \times 70 \times 10^9}}$$

$$= 0.021\ m.$$

$\therefore$ area $= \pi R^2 = 0.0014 = 1430\ mm^2$ (cf $32\ mm^2$ before

$\therefore$ mass increases by $\frac{1430}{32} = 44.7$ Now ~~weights~~ weighs

$0.29 \times 44.7 = 13.0\ kg!$