**Stokes drift**

Let us consider the tracer dispersion in a re-entrant channel bounded by walls at $y = 0, 1$. For simplicity we limit the study to a two dimensional flow. There is no mean flow and the turbulence is in the form of a traveling wave of small amplitude $\epsilon$,

$$\psi = \epsilon \sin[\pi(x-t)] \sin(\pi y), \qquad (u', v') = (-\psi_y, \psi_x). \tag{1}$$

The equation for the displacement $\boldsymbol{\xi}$ is given by,

$$\frac{\partial \boldsymbol{\xi}}{\partial t} = \boldsymbol{u}' + O(\epsilon^2). \tag{2}$$

The diffusivity is,

$$\begin{aligned} K_{ij} &= \begin{pmatrix} \overline{u'\xi} & \overline{u'\eta} \\ \overline{v'\xi} & \overline{v'\eta} \end{pmatrix} \qquad (3) \\ &= \frac{A^2\pi}{4} \begin{pmatrix} 2\overline{\sin[2\pi(x-t)]\cos^2 \pi y} & \overline{(1-\cos[2\pi(x-t)])\sin(2\pi y)} \\ -\overline{(1-\cos[2\pi(x-t)])\sin(2\pi y)} & 2\overline{\sin[2\pi(x-t)]\sin^2 \pi y} \end{pmatrix} \\ &= \frac{A^2}{4} \begin{pmatrix} 0 & \sin(2\pi y) \\ -\sin(2\pi y) & 0 \end{pmatrix} \end{aligned}$$

The Stokes drift in two dimensions is given by,

$$\bar{u}_S = -\frac{1}{2}\partial_y(D_{xx} - D_{yy}) = \frac{A^2\pi}{2}\cos(2\pi y), \tag{4}$$

$$\bar{v}_S = \frac{1}{2}\partial_x(D_{xx} - D_{yy}) = 0. \tag{5}$$